WHY WE LIVE IN $3+1$ DIMENSIONS

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(This work was initiated originally by one of us (CC), with Devashis Banerjee and Miguel Cardenas in Trieste during the summer of 98.)

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(Dedicated to Ilya Prigogine and John Archibald Wheeler, two mavericks of our time)

Abstract

In the same fashion that Einstein's General Relativity required Riemannian Geometry for its foundations, the New Relativity Theory requires a Cantorian-Fractal Spacetime, which is an example of Von Neumann's Noncommutative Geometry. Starting from first principles, an ensemble distribution for all the infinite dimensions of quantum spacetime is derived that requires Fermat's last theorem. An explicit computation of the average dimension yields a value very close to the exact number $4+\phi^3$, obtained previously from the Cantorian-Fractal Spacetime Model. The quantity ϕ is the Golden Mean. The Cosmological " constant problem " is never an issue within the framework of the New Relativity, since the Universe automatically self-organizes and self-tunes itself according to the Renormalization Group flow with respect to a local scaling microscopic arrow of time. The world began as a result of a non-equilibrium process of self-organized critical phenomena due to vacuum fluctuations in Cantorian-Fractal Spacetime. It is proven why we live in a metastable vacuum and are flowing to the RG fixed point of $D = 4+\phi^3$. A new phase transition will then drive the Universe to a quasi-crystal phase of lower average dimensions given by ϕ^3. Finally, we write down the Unique Quantum Master Action functional for the world in **C**-space, (outside spacetime) , that governs the quantum dynamics for the creation of spacetime, gravity and all of the fundamental forces in Nature.

1. Introduction

Recently we have proposed [1-5] a New Relativity Theory, where all dimensions and signatures of spacetime must be treated on the same footing, as the underlying foundations of M theory. So far it has allowed us to derive, on first principles, the stringy uncertainty relations and corrections thereof. The relationship between the Regge behaviour of the string spectrum with the area quantization. In particular, the full blown (infinite-dimensional) quantum spacetime uncertainty relations have been derived also , that include all the contributions of all p-branes. From $p = 0, 1, 2....\infty$. It was shown in [4] that there are no **EPR**, nor Black-Hole Information Loss, paradoxes in this New Relativity. More recently , it has allowed one of us [5] to derive the black-hole area-entropy relation in any dimensions, in two-lines. The New Relativity, based on Nottale's orginal Scale Relativity [6], postulates the Planck scale as the minimum distance in Nature, in the same vein that the speed of light was taken as the maximum (invariant) speed in Einstein's motion Relativity. The New Relativity requires an infinite-dimensional quantum spacetime. And such spacetime is provided by the transfinite continuum of Cantorian-Fractal Spacetime [7-9].

In the same fashion that Einstein's General Relativity required Riemannian Geometry for its foundations, the New Relativity Theory requires a Cantorian-Fractal Spacetime, which is an example of Von Neumann's Noncommutative Geometry. Starting from first principles, in section **3**, an ensemble distribution for all the infinite dimensions of quantum spacetime is derived that requires Fermat's last theorem.

An explicit computation of the average dimension in section **4** yields a value very close to the exact number $4 + \phi^3$, obtained previously from the Cantorian-Fractal Spacetime Model. The quantity ϕ is the

Golden Mean. We proceed to prove why we perceive 3+1 dimensions as a result of a coarse-grain long-distance averaging effect of the underlying Cantorian-Fractal Geometry.

We review in section **2** the two-line derivation of the Black-Hole Area-Entropy relation in any dimension and prove why the linear Bekenstein-Hawking relation is not valid in general.

In section **5** it is shown why the Cosmological " constant problem " is never an issue within the framework of the New Relativity, since the Universe automatically self-organizes and self-tunes itself according to the Renormalization Group flow with respect to a local scaling microscopic arrow of time.

An alternative cosmological scenario to the Big Bang, Inflationary models and the Variable Speed of Light cosmologies is proposed. The world began as a result of a non-equilibrium process of self-organized critical phenomena due to vacuum fluctuations in Cantorian-Fractal Spacetime. It is proven why we live in a metastable vacuum and are flowing to the RG fixed point of $D = 4 + \phi^3$.

In section **6** we discuss why a new phase transition will then drive the Universe to a quasi-crytal phase of lower average dimensions given by ϕ^3. Using Nottale's Scale Relativistic corrections we provide with two integral expressions that determine the values of the upper limits of the Hubble radius and the size of the quasi-crystal phase, implicitly and solely in terms of the Golden Mean ϕ.

Finally, in the conclusion , we summarize our results and write down, for finite values of D, the Unique Quantum Master Action functional for the world in **C**-space, (outside spacetime) , that governs the full quantum dynamics for the creation of spacetime, gravity and all of the fundamental forces in Nature. The quantum symmetry is given by a Braided-Hopf-Quantum-Clifford algebra. The QFT of such action is currently under investigation [17]. The quantum field was coined the ' Cantorion " in [9], whose vacuum fluctuations may have generated spacetime, gravity and all the forces that live in spacetime, in the large distance limit compared to the Planck scale.

In the $D \to \infty$ limit, the master action is associated with a topological Chern-Simons **p-brane** Field Theory, that should admit for its quantum symmetry extension, the infinite dimensional extension of a quantum W_∞ algebra; i.e it is a quantum infinite-dimensional-loop algebra that roughly spaeakin may be be visualized as the infinite colour (large N) limit of a quantum W_∞ algebra : a W_∞^∞ Berdashky type algebra. Some final remarks about the relation to the Weyl-Wigner-Groenewold-Moyal-Fedosov deformation quantization of classical phase spaces are given.

The essence of this work is based on the Renormalization Group techniques of QFT within the context of fractals. This picture of the infinite spiralings and infinite unfoldings of the hierarchy of infinite-dimensional worlds was something that the Renormalization Group was telling us all this time. One can focus our eyes to a small region of our screens. Lets label that region by a " point ". Certainly we can imagine " being inside that point ". To do it physically is another story. We require lots of energy to probe smaller distances. As one pumps in **energy** to " dive " towards that " point "; i.e to **zoom** into that point, we will be starting the unfolding of dimensions infront of our eyes (dimensions are energy are information). We will prove in section that by diving/spiraling into " smaller worlds " by pumping energy is tantamount of reversing the " scaling arrow " of the Renormalization Group Flow.

Energy is Information, hence by pumping energy inside a localized region we are creating information and thus " reversing " the local microscopic scaling arrow of time. In this case, we would be spiraling into another **different** hierarchical family of the Cantorian-Fractal Matryoshaka-like spacetime whose (local) average dimensions would be **greater** than $D = 4$: these would be the family that would belong to " our past " from the local scaling RG arrow of time. To reverse the RG flow arrow of time of the whole (global) Universe is a totally different enterprise. The Energy involved in achieving such a feat would be enormous : of the order of the total mass-energy of the whole universe.

Later on we will briefly speak of Nottale's " small/large distance duality " in Scale Relativity and how is tightly connected to the T-duality in string theory and the Ultraviolet/Infrared Entanglement in Noncommutative Geometry. Kreimer [11] has provided us with ample proof that there is a Hopf algebra associated with the Feynamn's diagrams, in the Renormalization and Combinatorial techniques of QFT. Within the multi-fractal/multivector/multiscaling view of the New Relativity, such rich mathematical machinery of Graph theory, Feynman graphs, Hopf algebras, Quantum Groups, Noncommutative Geometry, Nonassociative Geometry,... is an algorithimc/algebraic/combinatorial/self-recursive way of **coding** the infinite unfolding/spiraling into the transfinite Cantorian-Multi Fractal Hierarchical Matryoshka worlds. The Euler Gamma function is self recursive : $\Gamma(n) = (n-1)\Gamma(n-1)$.. Gamma functions have been essential

tools in the Renormalization process of QFT.

To support what we are saying we have Zamolodichkov's central charge theorem. It originated with flows of Conformal Field Theories in $D = 2$. It can be generalized to $D = 2n$. The central charge $c[g^i, \mu]$; g^i are the world sheet couplings; μ is the scale, substraction point; monotonically decreases along its RG trajectory towards the **infrared** :

$$\frac{\partial g^i}{\partial ln\ \mu}\ \frac{\partial c[g^i, \mu]}{\partial g^i} = \frac{\partial c[g^i, \mu]}{\partial ln\ \mu} \leq 0. \tag{1}$$

In string theory, CFT, central charges are directly related to **dimensions** . Critical Strings are anomaly free in $D = 26$ to cancel the $c = -26$ charge associated with the ghosts. Critical superstrings are anomaly fres in $D = 10$; etc.... Hence, central charges are directly related with dimensions. Zamolodichov's theorem is compatible with our assertion that as we " reverse " the " scaling RG" flow we are re-entering to regions of higher central charges and hence higher dimensions. For a profound connection between CFT and the Monster Group, Sphere packings, etc...we refer to [12]. Roughly speaking , the Monser Group, associated with the Golay error correcting code for a self-recursive system, (information and graph theory) is what codifies the unfolding/spiraling process into higher dimensions. Or conversely, by reversing the scaling arrow of time, the Monster is what codifies the condensation of dimensions from higher intial dimensions (on the average) to smaller and smaller ones (on the average) The Monster codifies the monotonic and gradual (quasi) crystalization process from higher to small dimensions. Finkelstein long ago had similar ideas about an underlying code for spacetime as a **q-network** of **q-processes** [35].

One does not have " a background spacetime " *ab initio*. Spacetime **emerges** as an Heraclitean process [13] : " One does not bathe twice in the same river ". Quantum Spacetime is truly a " process in the making " [14]. Gravity is a **nonlinear** theory. One has truly a nonlinear complex dynamical system that is able to self-tune itself like Biological systems do in Nature [16]. To put it up front : The Universe is an "Organism" or the Ultimate Quantum Machian computer. It is the relationships/interconnections : algebraic operations, among its representative elements (monads) that govern its evolution [13, 15]. Algebraic-Categorical Relationships are the only meaningful statements one can make in this New Machian Relativity [1-4, 54].

The Ensemble (of Universes) constructs its own Hilbert space of states as it evolves. At any given moment of " scaling time " (related to the Renormalization Group flow) the Universe is in a particular state with a particular dimension **on the average** that later will change with " scaling time ". Finkelstein has called this process a **variable q** law since one does not wish to include non-linearities in Quantum Mechanics. This is still an open question. For this reason, this view of the world does no allow for such statements like : " the cosmologically constant problem ". The non-linear self-tuning takes care automatically of this " problem ". One has a non-equilibrium statistical mechanical process of self organized criticality as Prigogine and collaborators have been advocating for quite some time.

We shall prove such results when we evaluate the (variable) average dimension and see explicitly that its value flows, in scaling time, with the RG flow to a critical " fixed " point value of $4 + \phi^3 = 4.236...$ Where ϕ is the Golden Mean. $\phi = \frac{\sqrt{5}-1}{2}$. The essential finding is that this " fixed " critical RG point is not a truly stable fixed point but a " metastable ". By Metastable we mean that is is not the true vacuum of the theory. The true vacuum corresponds to an average dimension of $\phi^3 = 0.236.....$ The Universe to-day is slowiy flowing to this metastable state of average dimension $4 + \phi^3 = 4.236.....$

2. A Two Line Derivation of the Area-Entropy Relation In Any Dimension

In this section we shall exhibit an example of the power behind the New Relativity. Based on a multi-vector/multiscaling/multi-fractal view of the world, that requires using Clifford Algebras, we shall review an " elementary " derivation of the Beckenstein-Hawking Black Hole entropy-area relation by one of us [5] to show the predictive power behind this New Relativity :

A Clifford algebra in D dimensions or degree D has 2^D independent " components " that represent precisely the point coordinates, holographic area, holographic volume, holographic four volume...coordinates associated with the hierarchy of point, **1**-loop, **2**-loop, ... histories, or excitations **OF** spacetime [1-4] . Therefore the number of "geometric bits " or degrees of freedom is : 2^D.

The **information** entropy in conventional Shannon's theory is just :

$$S = Kln\ 2^D = KDln\ 2\ \Rightarrow D = \frac{S}{Kln\ 2}. \tag{2}$$

We will see below that the $(D-1)$-dim hyper-area enclosing a D dim hypervolume (bulk) associated with a D-dim sphere of **unit** Planck radius is given by :

$$V_D = C_D R^D. \quad C_D \equiv \frac{\sqrt{\pi}^D}{\Gamma(\frac{D+2}{2})}. \quad A_{D-1} = DC_D R^{D-1} = \frac{dV_D}{dR} = DC_D 1^{D-1}. \Rightarrow D = \frac{A_{D-1}}{C_D 1^{D-1}}. \tag{3}$$

Therefore plugging in the value od D obtained in the last equation into the previous one we reproduce the entropy-area-geometric bits connection :

$$S = \frac{A_{D-1}}{C_D 1^{D-1}}\ Kln\ 2. \tag{4}$$

This means that entropy is given by area in Planck units times $ln\ 2$ times Boltzman's constant. It is **essential** to emphasize that one is **not** saying that we have a **linear** relation between entropy and area. like Bekenstein and Hawking assumed. The " constant " C_D is dimension dependent. There is only an approximate " linear " entropy-area dependence in a very narrow region of dimensions, roughly between $2-5$, when $R = \Lambda$. However this is not the case for higher dimensions !

Let us express $D \equiv (S/kln\ 2) = s$. Then $D \equiv s$, plugging this relation into (3) gives upon setting $R = \Lambda = 1$:

$$A_{D-1} = D\frac{\sqrt{\pi}^D}{\Gamma(\frac{D+2}{2})} = s\frac{\sqrt{\pi}^s}{\Gamma(\frac{s+2}{2})}. \tag{5}$$

Despite the complicated $Area$ versus $s = (S/kln\ 2)$ relation (5) one can immediately graph the inverse relation $s = s(A)$ in **two** steps :

1- simply by plotting the graph $Area$ versus D.

2- identify $D = s = (S/kln2)$ and rotate the graph counter-clockwsise 90 degrees

Upon looking through the transparent paper sheet, we will automatically have the graph of $s = s(A)$ and immediately conclude **two** things :

Bekenstein-Hawking conjecture that entropy is **linearly** bound by area is **not** true for every dimension. It is only true in a very narrow region of $D = 2$ to $D = 5$. When $R = 1$. For dimensions outside this range one clearly can see a departure from that linear relationship. !

For any given radius, one can follow the same procedure. We will notice that the linear area-entropy relation is valid for the same range of dimension but for a **different** range of areas. Once again, outside that range there is a departure from the Bekenstein-Hawking linear relation. For a given value of $D \equiv s$, as one increases the radius, the area tends to increase. This implies then that when we rotate the graph counter-clockwise by 90 degrees, we will see a linear relation between $s = s(A)$ for the same values of $s \equiv D$ but for larger values of area (new horizontal axis). One can verify that one does not violate the entropy Bekenstein upper bound of $(1/4)A$. The slope of that limited linear region, $(1/6.9)$ is less than $1/4$. Therefore, the entropy does not exceed the Bekenstein bound.

What does all this have to do with macroscopic black-holes ? The answer lies simply in mini-black-holes. A macroscopic black-hole is built from many " bits " or mini-black-holes of unit Planck radius. The state of a macroscopic black-hole is built by " counting " the number of micro-states accesible to the macroscopic system. Since Bekenstein-Hawking believed that there is a true linear relation between area-entropy (which we now know is only valid in a certain regime of dimensions and area), they could count the entropy by simply "adding" chunks or bits of areas : a linear superposition of the fundamental mini-black-holes states was possible due to the (limited) linearity of the area-entropy relation . A more fundamental way of combining bits of information-entropy is the following :

Suppose we wish to compute the entropy bounded by our cubic room. The way to do it is as follows : Since a cube is topologically equivalent to a sphere, simply deform the cube into a sphere in such a way that its area is **preserved**. Meaning that the volume is increased. Spheres have the geometric property

that for a given volume they minimize the area. And conversely, for a fixed area they maximize the volume. Evaluate the surface area of the sphere (equals the surface area of the cube) suitable normalized in Planck units. Take a sphere of unit radius (in Planck units) , evaluate its surface area and divide the large area of the sphere by the small area of the fundamental unit sphere and multiply the final ratio by $kln\ 2$, and the result will furnish the information-entropy of the big cubic room.

One will immediate say, this sounds fine but there are two main problems :

1- How are we going to pack all those small unit spheres into the big sphere without leaving any voids ? Simply deform each unit sphere into a fundamental small cube by preserving the area (and decreasing the volume). Deform the large sphere back to the intial cubic room, by preserving the area (while decreasing the volume). Now we can nicely stack together the small cubes against each other without leaving voids and pack the room tightly.

2- What if the information-entropy, given by the ratio of areas, in units of $kln\ 2$, was not an exact integer ? There is going to remain a small region left without being filled up. This is where the power of fractal geometry comes into play : Simply pack the room with fractal small spheres/cubes. In this fashion, due to the space-filling property of fractals we can pack the room without leaving any voids whatsoever.

Now we can understand why information charges flow to the boundary like a conductor [19]. As we begin to stack the fundamental small cubes together, on their common adjacent boundary, information will be squeezed. Since the New Relativity is based on the principle that the Planck scale is the minimum distance in Nature, this entails that it is not possible to squeeze information in that common boundary of the two adjacent cubes. One will have two bits in one unit of Planck area. This is not possible : one can only have one bit per unit Planck cell. Therefore, information charge will flow to the resulting boundary, after the cubes are stacked together. Repeating this process, over and over again, we will finally have that all the infomation must reside on the outer surface of the big cube. For further connection between the Cantorian-Fractal Spacetime to the theory of superconductivity and other phenomena see [19]. The fact that information cannot be packed. compresed beyond the Planck cell limits, is also related to the holographic origins of Chaos in $M(atrix)$ theory [20]. A characteristic signal of chaos is that one cannot pack the energy levels (spectra) into small regions of very high energy (spectral) density distributions. The repulsive (holographic) feature of these spectral lines is a signal of Quantum Chaos.

Since $SU(\infty), W_\infty, W_{1+\infty}, ...$ are area-preserving diffs, it is not surprising why they play such a role in Maldacena's $bulk(gravity)/boundary(CFT)$ (holographic) duality conjecture. For a direct connection between p-branes and the Moyal deformation quantization of the Lie algebraic structures of $SO(4k)$ Generalized Yang Mills (GYM) theories, which is **not** the same as the standard Moyal quantization program of the classical field theory of GYM, we refer to [18]. It is very unfortunate that many have confused the Moyal deformation of the $SO(4k)$ algebraic structures with the actual Moyal quantization of the classical GYM field theory. They are not the same thing.

p-branes were shown to be directly related to Composite Antisymmetric Tensor Field theories of the volume-preserving diffeomorphism in [23]. The relevance of this reformulation of p-branes as CATF, was that S and T duality were already built in from the very start : at the classical level. There was no need to conjecture S and T- duality : they are already built in. It is not necessary to probe into the non-perturbative quantum level. These composite antisymmetric tensor field theories (CAFT) of the volume-preserving diffs in [23], that were developed by Guendelman, Nissimov and Pacheva [51] , had the unsual property that they are examples of **Nonabelian** gauge theories which are **not** of the Yang-Mills type ! This was due to the composite (bootstrap) nature of these unusual field theories. In [1] we referred to the natural Duality among all p-branes and these CATF theories as Branes/CATF Duality. The natural deformation quantization of these theories is given by the Zariski deformed product associated with the deformation of the Nambu-Poison structure (the Jacobian) intiated by Bayen, Flato, Fronsdal, Lichnerowicz, Sternheimer and Takhtajan [49] . Kontsevich later on extended this deformation quantization to Poisson Manifolds [50] . It is clear from thse results that geometric quantization metods, multi-symplectic geometry, Jet Bundles .... are tightly connected with the deformation quantization of p-branes within the context of higher dimensional loop spaces. What remains to be done is to have a rigorous deformation Quantization formulation (a la Fedosov) of the multi-symplectic structures in these higher-dim loop spaces : W geometry [43].

The New Relativity furnishes the true quanta **OF** spacetime and have nothing to do with the background dependent spin 2 graviton. This corresponds to a linearized theory about an asymptotically flat background.

The New Relativity based on Cantorian-Fractal Spacetime is truly a background independent formulation of Quantum Gravity. Classical Spacetime does **not** exist *ab initio* : it is a quantum process in the making [14] , that in the long scale limit, yields the classical General Relativity and the Classical Riemannian Geometry. For this resaon it is not correct to assume that the classical symmetries, like Lorentz invariance, hold at this fundamental level : they must be replaced by Braided Hopf algebras/Quantum Group Symmetries, as so many authors have emphasized, in particular Mahid and collaborators. In the conclusion of this work we will explicitly write down the unique interacting Quantum Field Theory Master Action functional for the world [17] that was based on [1] and the revised version of [3]. Such action does not live in ordinary spacetime. It lives in **C**-space as a truly background independent formulation should be. It is nothing but a Clifford-algebraic generalization of Penrose's Twistors, with the inclusion of a Braided-Hopf-Quantum-Clifford Algebra as its quantum group symmetry.

To sum up : the New Relativity requires a Cantorian-Fractal Spacetime as its underlying Noncommutative Geometry. The goal of such theories is to derive " everything " from pure geometry as Einstein intended. Had he believed in the power behind Von Neumann's Geometry, which is the precursor of Noncommutative Geometry, he would have (most probably) changed his mind about Quantum Theory [6]. For a geometric unification of all fundamental forces within the framework of the Cantorian-Fractal Spacetime see [9].

In general, the graph $S = S(A)$ has a maximum value for the **slope**, which physically means (in the positive dimension region) that there is a maximum as to how much information (entropy = energy) one can **pack** per unit area ! The slope hits infinity signaling a phase transition : when the dimension goes to infinity (a phase transition occurs, spacetime evaporates leaving behind a fractal Cantor Dust, the area goes to zero so information density blows up). For radius of unity, the slope hits infinity between $D = 4$ and $D = 5$. This is the metastable RG fixed point of $4 + \epsilon = 4 + \phi^3$. Not suprisingly, this is another physical reason why we live in $D = 4$. The $E_8 \times E_8$ are natural ingredients in the Conway Sphere packings problems [8]. Can we reach $D = \infty$? In [4] we have discussed why dimensions, energy, information (DEI) are indistinguishable at the limiting Planck scale in Nature : the Planck scale. Once Nottale's full scale relativistic corrections, to the physical resolutions that an apparatus can resolve, are properly taken into account, it will cost us an infinite amount of energy to reach the Planck scale [5].

Imagining that one could, would mean than one would be able to plunge into the transfinite infinite abyss and " dive " though the uncountably infinite Cantorian-Fractal Spacetime transfinite continnum of uncountably infinite dimensions : As we reach what we think is the final end to our journey : " a point " , it will suddenly open up into a four-dimensional sphere. This process occurs an infinite-fold number of times and, over and over again, we will be inmersed into these infinite hierarchy of spirals [6]. One will never reach the Planck scale like a massive object cannot attain the speed of light.

Having taking this detour of energy, information, entropy, dimensions, RG flow....we believe that we have fulfilled John Wheeler's visonary dream that at Plank scales, dimensions, energy and information (DEI) are indistinguishable. In [4] we coined the word **hyperpoint** to represent the infinite-dimensional analog of a point, where **infinite** Dimensions, **infinite** Energy and **infinite** Information merge and become **ONE**. The hyperpoint is an **infinite** dimensional hyper-sphere of **unit** Planck radius and **zero** volume/measure. Even for an arbitraty large but **finite** radius, let us say the Hubble radius of 10^{61} planck units the volume will shrink to **zero** in the $D = \infty$ limit. We will explain below that this is due to a **phase** transition that causes spacetime to evaporate into a Cantor-Fractal Sea of Dust. And for this reason the notion of " distance " , " metric " disappears, because spacetime truly has evaporated, leaving behind only TOPOLOGY.

This " elementary " derivation of the Black Hole entropy based on Shannon's Information theory and **C**-spaces, Clifford-manifolds, is a direct result of the New Scale Relativity [1-4] Theory. It explains, in a straighforward fashion , the origins of the holographic principle by intoducing the holographic coordinates, that coupled to the elements of the Clifford algebra, which led to the full derivation of the blown generalization of the string uncertainty relations, including the contributions of for all p-branes [2,3]. This cannot **not** be considered as just a **numerical** coincidence : We are getting the **holographic** principle for **free** ! , it is a direct result of the Clifford algebraic multi-vector/multi-scale/multi-fractal Cantorian-Fractal view of the world advocated earlier [1-9].

We know of no derivation of the Black Hole entropy, in **any** dimension, in two simple lines, as far as we know. In the next section, we can clarify what we mean by the " walls " of the thermal reservoir being " a horizon ". But using exactly the same arguments that Planck used a century ago, to explain

the physical experiments of his time, when he derived the Black Body distribution, we have will arrive at similar conclusions for the quanta **OF** spacetime. Instead of photons, we have the infinite quanta **OF** a truly infinite-dimensional quantum spacetime, quantized in discrete geometric bits of point, **1**-loop, **2**-loop ....p-loop **histories** , evolving in the self-referential/self-supporting/bootstrapping bath of quanta as the true " dynamical quantum spacetime in the making ". A plausible experimental signal is nothing but the COBE data : the Black-Body distribution associated with what was believed to be remnants of a Big Bang : we will see that the New Relativity provides with a different cosmological scenario than the standard Hot Big Bang, Inflationary models and Variable Speed of Light Cosmologies.

3. Distributions : Fermat's Last Theorem Requires a Multidimensional World

The first thing to formulate the problem of how to compute average dimension is to find out what is the appropriate ensemble distribution density to use. To do this we need to define what one means by a " vacuum " and by "ensemble ". To find out what "vacuum" means, we proceed in the same fashion when one defines the zero point energy of the harmonic oscillator to be $\frac{1}{2}\hbar\omega_o$. Where ω_o is the fundamental frequency of the system. Once we achieve this goal then we can proceed like Planck did almost a century ago and view the world as one member representative of an ensemble comprised of an arbitray number of hyper-spheres of arbitray volumes and dimensions, but whose radii are **integer** multiples of the fundamental **quantum** of length : the Planck scale.

Since symmetry is a fundamental guiding principle in Phyiscs, we choose to have spherical symmetry to simplify our calculations. And also, because we have discovered, below, that there is Renormalization Group Flow Process that drives the system to average dimensions of lower and lower dimension to reach the observed four dimensional world we live in. Since The Renormalization Group Program requires a natural point or **subtraction** scale, μ, it makes sense to have a one-to-one correspondence between the radii of the hyper-bubbles, and the RG scale μ. Had one chosen hyper-cubes, instead of hyper-spheres, then additional length parameters would appear : length, width, height.... It is true that one could choose all of the sides of the hyper-cubes to have equal length, $a = 2R$ and still maintain the one-to-one correspondence. We are forced to assume that **topology**, in conjunction with radial symmetry , must play an essential role in the preparation of the ensemble. . A hyper-cube of dimension D has the same topology as a hyper-sphere of dimension D, although it has not the same geometry. We choose spheres for simplicity sake, and in **doing so**, we will end up face to face with Fermat's last Theorem, below. Hence, **number** theory plays a crucial role in the formulation of the ensemble of universes.

The ensemble of " spherical quantum oscillators ", universes, is in contact with a thermal bath, that the " Creator ", " Universal Observer", prepared intially at the Planck Temperature, T_P. The family of universes, hyper-spherical bubbles, can exchange energy with the " walls " of the " container ", , the thermal resrvoir, at $T = T_P$. We will call such " walls " of the container **the universal horizon** for reasons that will become clear in the text. Following Planck, we assume that the energy exchange occurs in **integer** discrete multiples of energy. Or what is equivalent; the hyper-bubbles of different dimensionality, can absorb quanta (from the reservoir) or emit quanta (to the reservoir) or exchange quanta among each-other. These " gravitons", or more precisely, " geometric bits ", are the true **quanta of spacetime** instead of the gravitons in conventional linearized gravity that are not truly background independent. This New Relativity furnishes the true quantum spacetime : a **q process** in the making quoting David Finkelstein. Spacetime is not assigned *ab initio* [14]. The classical spacetime that we perceive with our senses is only a long distance (effective theory) effect. The true quanta of length, area, volume, four-volume,—p-volume are discretely quantized in Planck units. We are just retracing Planck's original argument . Instead of dealing with " photons " we are delaing with "geometric " bits of length, area, volume and so forth.

One of the postulates of the the New Relativity Theory , which is based on nothing else but Chew's Bootstrap idea, is that **all** p-branes as made out of each-other. This is the essence of **duality**. This is the condition that one is using in this work : Quantum Spacetime is quantized in discrete bits of length, area, volume....We procced now how to formulate a **background independent** formulation of Quantum Grvaity.

Since these are the **quanta** or fundamental excitations **OF** spacetime, one does **not** have excitations of (spherical) p-brane fields propagating **in** an embedding taget spacetime background. Our ensemble is **self-referential** or self-supporting; i.e it obeys the bootstrap condition. We have a **background** independent formulation in the sense that one is not fixing, a priori, the target spacetime background. It is

arbitrary at this moment. The ensemble "creates" (self-tunes) its own background as it evolves. Smolin and Kauffman emphasized the possibility that self-organized-critical phenomena [16] may have in the description of Quantum Gravity. This will allow the universe to self-tune its fundamental constants. In particular the cosmological "constant ".

The self-referential/self-supported/bootstrapping background is "packed" with spheres of arbitrary dimensionality, and radii, exchanging energy with the " walls " of the thermal reservoir : the universal horizon. i.e. the "quanta" or excitations of spacetime, the analog of photons in Planck's original derivation of the Black Body distribution, in this case act, precisely , as their very own background , in which the hyperbubbles (p-branes) themselves propagate. To assert the truth or falseness of the assertion that we have a self-referential ensemble, we will invoke Godel's incompleteness theorem : to verify or deny our assertion one will be forced to ask the " Universal Observer " whether our proposition is true or false. Clearly, this is something we cannot do.

For the crucial role that self-referential noise and Godel's incompleteness theorem has in in the description of reality, in connection with Liebnitz ideas of *monads* we refer to [21]. Based on Godel's incompleteness theorem we can assert that the ensemble is **self** referential and self-organizing (self-tuning). This is another physical interpretation why we cannot ever escape ouside the universal horizon of the infinite-dimensional quantum spacetime. The walls of the hyperbubbles will have to move faster than light. Quantum Tunneling from the inside of the universal horizon to the outside is not possible because quantum spacetime simply does not exist "outside" the horizon. The Universal Observer is truly beyond us, beyond quantum-space-time, he lies at he hyperpoint. This is the physical implementation of Godel's incompleteness theorem which in essence says : " absolute truth is impossible ", only relative truths can be known. This fits correctly within the realm of the New Relativity and Cantorian-Fractal Spacetime.

To sum up : the " quanta " are the quantized (discrete) point, **1**-loop, **2**-loop, **3**-.....**p**-loop **histories OF** qunatum spacetime in Planck units. The " Universal Observer " lives outside of this infinite dimensional quantum spacetime, in **C**-space , which is a Clifford Manifold whose Clifford-algebra valued elements, **X** encode the point, loop, **2**-loop, **3**-loop .....histories excitations **OF** spacetime. . The ordinary spacetime coordinate x_μ linked to a point-history excitation, couples to the Clifford algebra gamma matrix γ^μ; the holographic area coordinates associated with the **1**-loop histories couples to the $\gamma^{[\mu}\gamma^{\nu]}$ and so forth : the holographic volume elements couple to the antisymmetric product of **3** gamma matrices..... . The **unit** matrix couples to the $p+1 = D$-dimensional (Lorentz scalar) worldvolume associated with the p- brane of maximal degree; i.e the spacetime filling p-brane that saturates the embedding dimension as it sweeps the (arbitrary) embedding D-dim space. The holographic coordinate $\sigma_{123.....D}$ associated with the maximal degree p-loop is the antisymmetric rank $p+1$ tensor, dual to the scalar world volume Ω_D in D-dimensions. It is just the top differential form. At the moment we shall not be concerned whether D is even or odd; i.e whether we ought to include the "γ^5".

Nottale's original Scale Relativity Theory and the New Relativity are not outlandish theories [52] . The New Relativity Theory is not due to a " close encounter of the infinite kind with extra-terrestials"; it is not due to a " visit from beings of the distant future " , nor is a result of " years of LSD psychotherapy with Stanislav Grof ", as it has been recently labeled by some members of the Scientific community [53] . It may in fact be true. Many Planets have already been found confirming Nottale' early predictions. Sadly enough (instead of being properly rewarded with increasing compliments) as the number of planets found confirming Nottale's predictions increases year by year, the number of insults increases as well [22]. Unfortunately this is something that the New Relativity nor the Cantorian-Fractal Spacetime Model will never be able to explain.

There has been a misconception of " the Universe " as being a single and fixed entity, instead of an **ongoing averaging process** of an infinity of universes belonging to the ensemble. The Universe we live in is not a fixed entity : It is just one representative element of an infinite ensemble of universes, going (evolving) through a continuous process and change : the Heraclitean view of the world [21].

This self-referential property, occurs because the universe is both open and closed at the same time. This simultaneous open/closed property is a hallmark of p-adic Physics and Non-Archimedean geometry [42]. In the latter p-adic Toplogies, all sets are both closed and open at the same time : they are called *clopen* sets . Even further, **every single** point belonging to each clopen set is a " center". All points are " centers ". This implies that there is **no** preferred bubble in the ensemble. All are centers, as a truly Machian Relativity ought to be.

To sum up, the quantum excitations **OF** spacetime in quasi-equilibrium with the thermal reservoir (horizon) act as their very own background where they themselves propagate. This confirms Chew's Bootstrap idea : the Universe is a Self-Supporting/Self-Referential entity of an infinity family of p-branes : all p-branes are made of each -other : this is our interpretation of **duality** [1,23]. We believe that this could be a truly **background** independent formulation of M theory. The quantum excitations **OF** spacetime (p-loop histories) comprised the very own background in which they themselves propagate. This is what one means by **self referential** noise as being a fundamental aspect of reality. We believe that Smolin and Kauffmann [16]....and others were right in following this train of thought.

So one of our most important conclusions of this work is that we agree with John Wheeler : **IT** from **BIT** : This **IS IT**. Or more precisely, **quantum IT** from **geometrical BIT**. Finkelstein and collaborators have been working along similar lines of using a Clifford algebraic approach to the **chronon** concept : discrete quantum " spinorial chess " units to describe the world, as fundamental building blocks of a **quantum process** and to formulate a **variable** quantum dynamical law. Our view is that this variable quantum dynamical law of Finkelstein and collaborators may be the one responsible for a self-organization process that varies from scale to scale. The world is **multi-fractal** [36] as a result of this variable **q** law. Life emerges as discrete jumps, from scale to scale, like phase transitions which occur in discrete jumps : a sort of bifurcation process. This is also related to Linde's view of inflation : the universe is an ever self-reproducing self-similar fractal process. Rigorously speaking we should say quasi-self-similar since the scales are not the same : We have multi-fractality. For recent work on the importance of a fractal-like evolutionary tree of life by Nottale and collaborators we refer to [24].

With this preamble we are ready to start by defining what we mean by " vacuum ". Firstly, we must specify what is the frame of reference with respect to which we define the "vacuum ". In [2] we have shown that because the cosmological " constant " is the Fourier conjugate variable to the Ω_D world volume, and since the latter, is just **one** component, of the many multivector components, associated with a multivector Clifford-valued object, then it is **not** truly a relativistic invariant (under the New Relativity) , under poly-dimensional transformations [25], or automophisms of the Clifford-algebra. What is truly an invariant is the **norm** of the whole multivector **K**, conjugate to the "coordinate " Clifford algebra valued multivector **X**. The cosmological " constant " is **not** a constant by **definition**. For example, the Energy in Einstein's Relativity is not a constant but is one of the components of a four vector. What is an invariant is the : $E^2 - \mathbf{p}^2 = m^2$. Exactly the same happens with the cosmological " constant ".

Hence to define a " vacuum " we must specify the frame of reference in the Clifford manifold, **C**-space. This reference frame is just " the box ", or thermal reservoir prepared by the " Universal Observer " living in **C**-space. Having chosen a given frame of reference, then we can now speak of what we mean by " lowest " energy. The analog of the zero point energy of the harmonic oscillator. Lets us take **all** the infinite families of the hyper-bubbles to have precisely equal radii of exactly one unit of Planck length, $R = \Lambda$. The volumes of the hyper-bubbles will vary from dimension to dimension according to the formula of volume of S^D as functions of D and R. The Planck scale is set as our basic reference (minimal distance in Nature); any excitations must be in discrete jumps of Λ. Since we have set the $R = \Lambda$ then we have attain what we mean by bubbles of minimum energy.

A question rises :

Do all the hyper-bubbles of Planck Radius have or not the same value of " lowest " energy ?

If one assumes that each bubble will have **different** "lowest" energies, according to their different dimensionalities, we will be contradicting ourselves. Because, by "lowest" energy, once a frame of reference has been chosen, we should mean the **same** lowest value in **every** dimension. Choosing different "lowest" values for the energy will explicitly break poly-dimensional covariance : **all** dimensions must be treated on the same footing. Hence, all bubbles of exactly one unit of Planck size must have the same value of lowest energy : $\mathcal{E}_{vac}$.

Therefore, the " radial-oscillators " energy excitations of the bubbles, in any dimension, are suitable power multiples of the fundamental vacuum energy :

$$E_D = \mathcal{E}(\frac{R}{\Lambda})^D = \frac{\mathcal{E}}{C_D \Lambda^D}\ C_D R^D. \quad V_D(R) = C_D R^D = \frac{\sqrt{\pi}^D R^D}{\Gamma(\frac{D+2}{2})}. \tag{6}$$

where we have written the volume of a hypersphere in D dimensions in terms of a D dependent constant,

C_D related to the Gamma functions and $\pi^{D/2}$, and its radius. We can re-write eq-(6) :

$$\frac{E_D(R_1)}{C_D R_1^D} = \frac{E_D(R_2)}{C_D R_2^D} = ... = \frac{\mathcal{E}}{C_D \Lambda^D}. \tag{7}$$

the last equation implies that bubbles of fixed dimension, D, for variable radius $R_1, R_2, ...$, have identical energy *densities* to those bubbles of Planck sizes in the same dimension. This is the reflection of " incompressiblity " or volume-preserving diffeomorphism symmetry that has been studied within the context of p-branes in many occasions. Bubbles are quantized in multiple units of Planck units. If a D-dim bubble size increases, by absorbing "bits " of potential energy, like an ordinary oscillator, it must be do so in orchestrated **discrete** Planck units to maintain, the original Planck size D-dim bubble, energy **density** invariant.

Nature follows the minimum energy principle. Since we have argued that all bubbles of unit Planck sizes have the same vacuum energy, in **any** dimension, equal to the $\mathcal{E}_{vac}$, and because the volume is D-dependent, varying from dimension to dimension, the ensemble will consist of different populations of bubbles of different dimensions. The volume distribution as a function of dimension, for fixed unit Planck ratio, has a volume **peak** in the $D = 4 - 6$ region. Meaning that these bubbles will have the **same** vacuum energy $\mathcal{E}_{vac}$, for **larger** volumes, and hence, **lower** energy densities than those bubbles in the extreme zero volume cases ($D = -2, D = \infty$). These extreme cases correspopnd to fixed energy but ∞ energy density due to the zero volume.

Therefore, we arrive at :

The ground state of the **initial** system consists of a **condensation**, at planck T_P, of an infinity of bubbles of all possible dimensions, where each single bubble has a radius of unit Planck size and constant energy $\mathcal{E}_{vac}$. Condensation at High temperatures have been studied by Rojas et al [26]. The view of the possible vacuum of Quantum Gravity as a condensate was explained to us by Chapline [27]. The condensation will occur such that the ensemble of bubbles will distribute itself so as to **minimize** the energy **densities** : For example : bubbles of $D = 4 - 6$ will have **higher** statistical weight than those bubbles in the the ends $D = -2,\ \infty$. Precisely, based on this energy-density minimal principle, familiar in fluid mechanics, we will take as our ensemble distribution of dimensions the Gamma distribution (for Planck radius that we can normalize/set to unity) :

$$C_D = \frac{\sqrt{\pi}^D}{\Gamma(\frac{D+2}{2})}. \tag{8}$$

the values of C_D yield precisely the statistical weights of the distribution of dimension. Later we shall include an energy dependence to the distribution so we can use a Bose-Einstein distribution to account for **thermal** effects : for slowly decreasing Temperatures , from the Planck regime to the **3** degrees Kelvin of today.

Now we can prove why Fermat's last theorem imposes upon us to choose **all** innfinite dimensions for our ensemble.

Upon collisons two bubbles, A, B can yield a third bubble C, like ordinary chemical reactions or the **3** point vertex of string field theory. . Let us assume for the moment that **all** the bubbles have equal dimension D. Our only assumption in this work is that the " chemical " reaction **conserves** energy, and that energy is additive , since the energy densities for **equal** dimensionality are constant, the energy density is an intensive quantity, versus the energy that is an extensive one. This is required of a thermalization process :

$$E_A + E_B = E_C \ \Rightarrow (n_A)^D + (n_B)^D = (n_C)^D. \tag{9}$$

where, after a cancellation of a common $\mathcal{E}_{vac}$ factor on both sides of the equation. , we have expressed the energies in suitable integer numbers (quanta) n_A, n_B, n_C accordingly in eq- (10).

Fermat's Last theorem, proven finally by Andrew Wiles, says that eq-(10) has **no** solution for **nonnegative** integers for integer dimensions larger than **2**. For example, there is no postive integer raised to the cubic power that can be expressed as the sum of the cubic powers of two positive integers. Only for integer $D = 2, D = 1$ there are solutions to the Diophante equations. (Pythagoras theorem for $D = 2$).

Since we live in $D > 2$, then Fermat's last theorem forces upon us a different energy balance equation than (10) :

$$(n_A)^{D_A} + (n_B)^{D_B} = (n_C)^{D_C}. \tag{11}$$

where not all of the dimensionalites D_A, D_B, D_C can be equal. Hence we have arrived at one of the most relevant conclusions of our work :

For equilibrium (or quasi-equilibrium), to be attained through a **thermalization** process of collisons, Fermat's last theorem requires that **dimensions** must change in the collisons. Since dimension is a topological invariant, the collision process provides us with the simplest example of a topology changing process. The geometry is fixed to spheres. Hyperbolic geometries, like the upper complex plane, de Sitter, Anti de Sitter spaces could be studied from another framework. The problem with these topologies is that they comprised spaces that are **open** and non-compact. One could compactified them by attaching the projective boundaries at infinity, that are topologically equivalent, in the Anti deSitter case, to spheres : S^n. This could be an interesting study case. For the time being we concentrate with compact worlds. Perhaps a phase transition will not only change topologies but also change one geometry to the other. For example, in $3D$ one has to include Thurston's 8 different geometries and matters get even more complicated. This discussion and the implication for the resolution of Poincare's unresolved conjecture is left for future analylisis.

An example of a dimension-changing process is :

$$(2)^1 + (5)^2 = (3)^3 \;\Rightarrow A(n_A = 2; D = 1) + B(n_B = 5; D = 2) \rightarrow C(n_C = 3; D = 3). \tag{12}$$

2 quanta (geometric bits) plus **25** quanta (geometric bits) yield **27** quanta (geometric bits). Information is conserved in this transition since Energy has been conserved. The reversed can happen also : C can break into $A + B$. This will be like a dimensional " compactification " process. A $D = 3$ bubble of radius 3 in Planck units, has "compactified " into a bubble of $D = 1$, of radius 2 in Planck units, and another of $D = 2$, of radius 5 in Planck units.

Collisions like $A + B \rightarrow C + D$ folow the similar argument. Using string theory we know that the four point vertex can be obtained by a **sewing** process of two three-point vertices. Interactions in string theory are mediated by topology : the interpolating world sheet. The four point vertex, at tree level, is usually described by a two-sphere with the four vertex operators, representing the on-shell emission, absorption of particles, attached to the four punctures. Higher loop corrections require to use Riemann surfaces of arbitrary genus and to perform the perturbation expansion. We do not have to be concern about this issue. Perturbation theory is not the right way to look at the world. One of the most famous examples of how wrong is to rely on perturbation theory is Gravity in $3D$. The conventional Einstein-Hilbert action in $3D$ is a perturbatively non-renormalizable quantum theory. However, this did not stop Witten to show that it could be quantized as a Topological Chern-Simons Field Theory.

Mainly because it is a well know theorem (to the experts) that there is **no** algorithm to classify topologies for $D \geq 4$, we have to abandon this naive perturbative picture for a non-perturbative one : Dimensions do not fluctuate midly : they fluctuate completely in Cantorian-Fractal Spacetime. This agrees with the program of Hull-Townsend-Witten M theory. Perturbative string theory could have not ever recaptured a dimension change from 10 to 11, let alone the inclusion of an infinity of dimensions. The first person who brought to our atttention that the world could be actually infinite-dimensional was the mathematician Javier R. Luque [28].

Therefore, when using **all** infinite dimensions, based on straightforward physical arguments and Fermat's last theorem, obviates the need to use perturbative methods and to worry about " summing " over all topologies. The thermalization process due to collisons and Fermat's last theorem automatically encodes " summing " over different topologies. A S^2 does no have the same topology as a S^4, simply because the dimensions are different !

Now, we are going to evaluate the average dimension of an statistical ensemble of bubbles of arbitrary dimensionality and radii in quasi-equlibrium with a thermal reservoir at a given temperature. Starting with Planck's T_P, we will show how any infinitesimal perturbations from the initial conditions, that we call the " balance " conditions, will tip and drive the system to a progressive increase of average dimension, until it reaches a peak average value, and then there is a monotonic decrease of both temperature and average dimensions, to lower and lower average dimensions and temperatures. The system will come back to the initial average dimension it started out with, but with a much lower temperature than Planck's T_P.

In a sense, any infinitesimal perturbations from the intial " balance " conditions will induce a microscopic Renormalization Group Flow arrow of local time [29]. Since Information is a measure of Dimension, Energy, Entropy, a decrease in the number of average dimension with respect to the RG local scaling " time " will signal a change of information entropy. The RG flow will roll towards lower average dimension, lower temperatures : i.e towards the infrared. This agrees precisely with Zamolodichkov's central charge CFT theorem.

Notice that the RG flow induces an arrow of scaling time that must not be confused with the coordinate time that we perceive with our clocks. We are speaking of two times. The scaling time is the universal local arrow of time [30]. There is no gobal time here. The other is the usual coordinate time. Bars's (and collaborators) S theory has also addressed the important issue of "two-times" in Physics. Also crucial to derive why we live in four dimensions is the role of **negative** dimensions. We have arrived at another very relevant conclusion of our work :

In this New Relativity, dimensions must be defined relative to a **zero** point dimension D_o. In the same vein that energy is defined in phyiscs. A careful look at the graph of volume vs. dimension, for finite radius, shows us that at $D = -2$ there is a pole in the gamma function that drives the volume to zero. We will take such value of $D_o = -2$ as the zero-reference dimension with respect to which we measure dimensions.

In a similar fashion, when $D = \infty$, for finite radius, the volume also goes to zero. This was the crucial condition that allowed us to prove that there **no** such things as the EPR paradox in the New Relativity. In the $D = \infty$ limit there is a phase transition [4] spacetime evaporates, it fractalizes into an spherical symmetric Cantor like fractal dust of **zero** hypervolume and hypersurface. Since spacetime has truly evaporated leaving behind the Cantor Set of fractal dust, the notion of " distance " among the members of the Cantor dust is meaningles. There is **no** distance and for this reason all the points are interconnected/entangled and information exchange bewteen two photons at the two opposite extremes of a line connecting the North and South Pole, is instantaneous [4]. All the poins of the Cantor Dust live inside the **hyperpoint** : the infinite dimensional analog of a point. A hypersphere of finite radius but **zero** size : we are imbuing a mathematical " point " with a true physical meaning.

This is the essence behind the construction of the Cantorian-Fractal Spacetime and its associated von Neumann Noncommutative Geometry [6]. One constructs $\mathcal{E}^{(\infty)}$, by taking infinite intersections of infinite unions. In this fashion one packs space-time as densely as possible, following the Maudlin-Williams Golden Mean theorem, with sets comprised of uncountable infinite number of fractal dust points. To see the importance on the role of the Golden Mean in Phyiscs, Renormalization Group and Average dimension we proceed to the next section .

4. Average Dimensions and Cantorian-Fractal Spacetime

4.1 Explicit Calculation of Average Dimensions

Before we begin the actual calculation of the average dimensions, we deem it important to recall Nottale's Scale Relativistic Corrections for a given D-dim volume sphere of a given radius. This will be essential when in the final sections, we will write down the expression that allows us to compute exactly the upper limits on the Hubble radius, before the Universe begins to " roll down " into a quasi-crystal phase whose limiting value of the (average) dimension is ϕ^3. Where ϕ is the Golden Mean. If one takes $D_o = -2$ as the zero reference dimension, the quasi-crystal phase is finally completed when $< D >= 2 + \phi^3$. There will be a drop of **four** dimensions, on average until the Universe reaches the end point of that quasi crystal phase. A quasi-crystal phase of enormous size, extremely cold temperature, not absolute zero nor zero entropy.

The scale relativistic corrections for fractal spheres are :

$$V = V_o \xi^{-D(\gamma(\xi)-1)}. \quad \gamma(\xi) = (1-\beta^2)^{-1/2}. \quad \beta^2 = \frac{ln^2\ (\xi/R)}{ln^2\ (R/\Lambda))}. \tag{13}$$

The resolution ξ ranges from the finest Planck scale resolution to the course-grain one of R. It is meaningless to say that we have a sphere of radius R if our apparatus cannot resolve such distances to begin with. When one hits the finest resolution in Nature, the Planck scale, $\xi = \Lambda$, $\gamma \to \infty$, $V \to 0$ meaning that the volume of the hypersphere is zero which is tantamount of saying that we have hit the **infinite** dimensional limit where spacetime evaporates into a sea of fractal dust, the **hyperpoint** . To achieve this one requires an **infinite**

amount of energy. The Planck scale is the natural Ultraviolet regulator in Nature, as it was also advocated by Thiemann [31]. .

The difference between the fractal (Hausdorff) dimension and the Topological dimensions are :

$$D_F - D_T = D(\gamma(\xi) - 1) \equiv D(\xi). \tag{14}$$

For the time being we will not include Nottale's Scale Relativistic Corrrections in our calculations, until we prove why Nottale's corrections are precisely those corrections that shift the average dimension that we get $D \sim 4 + \phi^2 = 4.38....$ to the exact value of $D = 4 + \phi^3 = 4.236...$ obtained from the Cantorian-Fractal spacetimes [6-9]. The fundamental reason is that we are going to evaluate the averages using smooth spheres. To be able to reach the exact Cantorian-Fractal Spacetime value of $4+\phi^3$, requires using packing spheres with the Cantorian-Fractal Dust. It is for this reason than Nottale's Scale Relativistic Corrections are crucial. This is could be a " numerical evidence" that Scale Relativity should be true, in addition to the confirmation of planets.

The average Dimension (without including fractal corrections) is given by :

$$< D >= \frac{\int_d^\infty dDdE\ D\rho(D,E)}{\int_d^\infty dDdE\ \rho(D,E)}. \tag{15}$$

with d being the lower limit of the integrals that we will take to be $D = -2$ for physical reasons explained earlier and next. The Energy of the bubbles is :

$$E_D = E_o(\frac{R}{\Lambda})^D. \quad dE = DE_o(\frac{R}{\Lambda})^{D-1} d(\frac{R}{\Lambda}). \tag{16}$$

where E_o is the vacuum energy and Λ the universal Planck length in every dimension, in units of $\hbar = c = 1$, is given in terms of Newton's gravitational constant $G_D, G_{D-1}, G_{D-2}.....$ in succesive $D, D-1, D-2....$ dimensions.

$$\Lambda = G_D^{\frac{1}{D-2}} = G_{D-1}^{\frac{1}{D-3}} = ... \tag{17}$$

In ordinary units in $D = 4$ the Planck scale is $10^{-33} cms.$ When $D = 2$ the value (17) is singular. In $D = 2$ the Einstein-Hilbert action is a topological invariant. We can select $G_2 = 1$ in (17) in such a form so $(G_2)^\infty = 1^\infty = 1$. Therefore, one may choose then the universal scale to be 1 in the units of $\hbar = c = G_2 = 1$.

Using a Bose Einstein distribution we have then :

$$< D >= \frac{\int_d^\infty dD\ D^2\sqrt{\pi}^D[\Gamma(\frac{D+2}{2})]^{-1} \int_1^{(R_H/\Lambda)} d(R/\Lambda)\ (R/\Lambda)^{D-1}\ [e^{\frac{E_o(R/\Lambda)^D}{KT}} - 1]^{-1}}{\int_d^\infty dD\ D\sqrt{\pi}^D[\Gamma(\frac{D+2}{2})]^{-1} \int_1^{(R_H/\Lambda)} d(R/\Lambda)\ (R/\Lambda)^{D-1}\ [e^{\frac{E_o(R/\Lambda)^D}{KT}} - 1]^{-1}}. \tag{18}$$

Performing the thermal integral yields :

$$< D >= \frac{\int_d^\infty dD\ D\sqrt{\pi}^D[\Gamma(\frac{D+2}{2})]^{-1} F(a, b^D)}{\int_d^\infty dD\ \sqrt{\pi}^D[\Gamma(\frac{D+2}{2})]^{-1} F(a, b^D)}. \tag{19}$$

where

$$a = \frac{E_o}{KT}. \quad b^D = (\frac{R_H}{\Lambda})^D. \quad F(a, b^D) = a(b^D - 1) + ln[\frac{e^a - 1}{e^{ab^D} - 1}]. \tag{20}$$

There are three special cases to study :

(i) When $b = 1$, meaning that the bubble radius does not grow beyond the intial Planck size, " mini-black-holes ", the function $F(a, b^D) \equiv 0$ and it decouples from the calculations due to an explicit cancellation between the numerator and denominator.

(ii) When a, b are very large, the $F(a, b^D)$ approaches zero asymptotically for any dimensions, and again, it decouples from the value of the average. The average dimension comes back to its original value with the main difference that the final temperature is much lower than the T_P, due to the fact that the radius of the bubble-universes are very large also, compared their original sizess (the Planck scale).

(iii) Due to the limiting cases in : (i), (ii) , we can infer (and verified by explicit numerical calculations) that when a, b lie in an intermediate regime, then $F(a, b^D)$ does contribute to the average dimension in a substantial way. Meaning that the infinitesimal fluctuations drive the average to higher and higher values, reaching a peak, and then it monotonically decreases, flowing asymptotically back to the RG metastable fixed point of $< D >= 4 + \epsilon = 4 + \phi^3 = 4.236....$ Order begins to emerge from Chaos as soon as the system begins its descent (in average dimension and temperature) back to the metastable RG fixed point (the world we live in).

Next, we are going to prove how important is ϕ , the Golden Mean in all of our discussions. Not only it will help us understand why we live in four dimensions but why do we have the signature of $3 + 1$, and why perhaps, it may be the deep underlying reason of chiral symmetry breaking in Nature !

4.2 $< D > = 4.236067977... = 4 + \phi^3$ as the Exact Average Limiting Dimension

Taking the full principle of Relativity to the fullest, from now on when we say D we mean dimension with respect to a zero point dimension D_o , so by D we mean $D - D_o = D'$. This will affect the lower limit in the integrals (18, 19). Now we have instead $D_{lower} - D_o = \epsilon$. The most natural value for the lower limit is $D_{lower} = -2$ since it corresponds to the pole of the gamma function in the distribution (it corresponds to a hypersphere of zero volume). Then our **new** lower D' limit is $-2 - E_o = \epsilon$. Choosing $\epsilon = 0$ yields that the zero point dimension is nothing but $D_o = -2$, the pole of the gamma function.

From now on we shall not write d for lower limit but $-2 - E_o = 0$ instead. By a close inspection, the average D' dimension (19), $D' = D - D_o$, can be re-written as the ratio of two averages :

$$< D'(t) > = \frac{< D' F(a(t), b(t)^{D'}) >}{< F(a(t), b(t)^{D'}) >} \ \geq < D' >_{lower} . \tag{21}$$

The lower bound in the average dimension $< D' >$ is the physical reflection that, intially, quantum dissipative effects (chaos) must *increase* the average dimension beyond the value $< D' >_{lower}$, *without* quantum dissipative effects. The expanding bubbles will not cool down below the Planck T_P due to these quantum-dissipative effects (chaotic phase). Once the average dimension reaches a peak value, the ongoing expansion of the bubbles will out-balance the re-heating due to quantum-dissipative effects, and slowly , the bubbles will begin to cool, and the average dimension will begin to roll down back to its original value. This will be the beggining of the " order " phase versus the initial chaotic phase (quantum dissipation). Once there is a trade-off of thermal effects for the initial quantum dissipative effects, one can verify that the average dimension cannot go below the metastable RG fixed point of $4 + \phi^3$ until we hit the phase transition point and roll down to the quasi-crystal phase (the true vacuum).

As we have shown in the two of the cases above, When $F(a, b^{D'}) = 0$ in eqs-(18,19), it will decouple from the average : it will cancel explicitly in the numerator and denominator. leaving an integral solely with respect to the dimensions. Taking for $\epsilon = -2 - D_o = 0$, as the lower limit of the D' integration, which implies that the zero point Dimension $D_o = -2$, yields a lower bound of the ratio of the two integrals in eq-(19), after the cancellation of the $F = 0$ critical cases, of :

$$< D' > = 6.3876 = < D - D_o > \quad \Rightarrow < D > = 4.3876 \sim 4 + \phi^2 = 4.3819..... \tag{22}$$

Now we shall look for a lower bound. Instead of performing the integral, and analytical continuation in dimension, one may compute the average dimension by a performing a discrete sum in the numerator and denominator, and take the subsequent ratio. The discrete average turns out to be, after performing the summation of 1200 terms, and after taking into account the zero-reference dimension of $D_o = -2$, of the order of $< n > (discrete) = 4.3267$ which is smaller (as expected) than the value obtained from the explicit integration : $4.3876 \sim 4 + \phi^2 = 4.3819...$.

These numbers based on a discrete and integral averages, agree very closely with the results of the transfinite continuum Cantorian-Fractal Spacetime Models of [6-9]. These Cantorian-Fractal models provide, due to the Maudlin-Williams Golden Mean theorem, with the densest packings and the **exact** value of the average dimension :

$$< D >=< dim_F\ \mathcal{E}^{(\infty)} >= \frac{1+\phi}{1-\phi} = \frac{1}{\phi^3} = 4+\phi^3 = 4.236067977. \quad \phi = \frac{\sqrt{5}-1}{2} = 0.618..... < \frac{ln2}{ln3}. \qquad (23)$$

Notice that the Golden Mean $\phi = 1/(1+\phi)$ is slightly smaller than the dimension of the middle segment Cantor set $ln2/ln3$.

The Golden Mean ϕ provides us with the basic "unit", or dimension-building block, where all the Sets of the Transfinite Cantorian-Fractal spacetime models are constructed : These are the densest spaces obtained by taking infinite intersections of infinite unions. Such spaces are the paramount example of the original Von Neumann Geometries which were the precursors to Connes Noncommutative Geometry. The value of $d_c^{(0)} = \phi = 0.618033984....$ is the fractal dimension of a physical structure living in a 0 topological dimension.

This in effect means than one is packing, compressing (let us say information as an example) the fractal dust points of dimensionality equal to ϕ into a " point ". We write a " point " to emphasize that due to the minimal Planck scale one really does not have " points" in Nature. A " point " is smeared out int a " fuzzy " ball , and for this reason the naive concept of having a **fixed** and well defined dimension is consequently **lost**. The "point" is smeared out into a fuzzy ball/set of all different topological dimensions, ranging from $-\infty$ all the way to $+\infty$. The former set : $\mathcal{E}^{(-\infty)}$ is called the **void** virtual set and the latter set $\mathcal{E}^{(\infty)}$, the Universal set. Their corresponding dimensions are inverse duals to each other : $d_c^{(+\infty)} = \infty = 1/d_c^{(-\infty)} = 1/0$.

For a modern application of Virtual sets of negative topological dimensions see Peter Aczel's Non-Well-Founded Set Theory [34]. Shortly we will discuss why these virtual sets of negative topological dimension carry negative information-entropy, or anti-entropy, as Conrad has called it [38]. This is nothing but the direct application of Dirac's ideas of negative energy, antimatter, to dimensions/information/entropy. To specify the true vacuum of the theory we must assume that the Dirac sea of topological negative dimensions, below the topological value of $D_o = -2$ is filled up. Otherwise we would encounter a situation where the Quantum Universe will cascade downwards and dissapear into the void set $\mathcal{E}^{(-\infty)}$. The void set is a structure living in topological $-\infty$ dimensions but having for fractal dimension 0; i.e the void set is comprised of one single fractal dust point. The Universal set, being the dual set, has an uncountable infinity of fractal dust points, for this reason its fractal Cantorian dimension is ∞. Cascading and spiraling downwards from the Universal Set to the Void set would have been a true ironic twist of fate : " from fractal dust to fractal dust.....that was the story of the Quantum Universe ...." .

The Penrose tiling is the best known representative of a zero dimensional Noncommutative Space. Since $D_o = -2$ is the truly zero-reference dimension, the Penrose Noncommutative space is, in fact, two-dimensional relative to the true " zero " $D_o = -2$ reference level. Not surprising , the Penrose quasi-crytal is nothing but the quasi-periodic tiling of the two-dim plane. The Golden Mean ϕ is an essential ingredient of the Penrose tile supported also by Atiyah's K theory arguments.

Hence the average dimensions provided respectively by the Cantorian-Fractal Spacetime Model, Discrete Sums average and the smooth integral average satisfy the following relation :

$$4+\phi^3 = 4.236... \ < 4.3267\ (discrete)\ < 4.3876\ (integral) \sim 4+\phi^2. \qquad (24)$$

The numbers come out to be pretty close, taking into account that we have used smooth spheres for the averages and that $4+\phi^3$ is the exact result obtained from the densest packing provided by the Maudlin-Williams Golden Mean Theorem.

There is another approximate value for the average dimension related to Knot theory and the Jones polynomial [6-9] $< D >= -2/ln\phi = 4.156...$ that lies in between $4+\phi^3$ and $4+\phi^4$. The Cantorian-Fractal model yields the exact value since it provides with the densest packing. For example, the Conway Leech lattice packing (The Leech lattice is a crucial ingredient in the $E_8 \times E_8$ heterotic string) , taking for upper limits the value $D = n = 24$ yields an average dimension of roughly $< n > \sim 4.07 - 4.09$. Such calculation requires weighting each sphere with the packing density number, $\Delta_n \leq 1$ that is roughly speaking given by the ratio of two measures : the volume of the (inscribed) n-dimensional sphere divided the volume of the embedding polyhedron in $D = n$.

We believe this are **not** just numerical coincidence. The exact value of the average dimension , $< dim\ \mathcal{E}^{(\infty)} > =\ d_c^{(4)} = 4+\phi^3$ is a signal of quasi-ergodicity [6-9] : when the topological dimension is of

the same order as the Fractal dimension . When $D_T > D_F$ it is a stable case. When $D_T < D_F$ it is ergodic. We are living in a **metastable** quasi-erogodic state or " vacuum ". The true vacuum, as we argued earlier coresponds to the dual inverse fractal dimension of $d_c^{(4)}$ equal to $d_c^{(-2)} = 1/(4+\phi^3) = \phi^3$; or $2+\phi^3$ relative to the zero point $D_o = -2$.

This is fine, and looks " pretty ", but how does this explains why we live in a $3+1$ dimensions ? This is related to the long-distance coarse-grain averaging process of an underlying fractal geometry :

The attempt to derive a probabilistic interpretation value for the spacetime dimensionality goes back to Wheeler in his formulation of Geometrodynamics. The New Relativity, in conjunction with the Cantorian-Fractal Geometry, confirms Wheeler's idea. In the final conclusion we will write down a Unique Quantum Master Action functional for the world. Such action lives outside spacetime [17] : it is a field theory that was originally proposed in Noncommutative **C**-spaces [1,3]. In this fashion, we are furnishing a truly background independent quantum field theory of gravity and all the other fundamental interactions : a Pregeometic Quantum Field Theory formulation of the world. We get everything from purely geometrical fluctuations. Classical spacetime and ordinary gravity emerge later, as a result of a symmetry breaking mechanism, due to the long-distance coarse-grain averaging process of a truly fractal geometry.

Boyarski and Gora [38] have argued that spacetime has a lattice structure which determines the probabilistic behaviour of a moving quantum particle. Such lattice structure arises from a coarse-graining averaging of the quasi-ergodic Cantorian-Fractal spacetime. The best analogy of this may be the lattice-like pattern which one obtains by averaging over large numbers of instantaneous chaotic images of the waves of say the Faraday instability , as has ben demonstrated by Gluckmann and Gollub.

Now we shall proceed to prove it :

The breakdown of poly-dimensional-isotropy due to the coarse-grain long-distance averaging process goes as follows : Initially, prior to the break-down of poly-dimensional-isotropy, one has on average four orthogonal dimensions; each of which could fluctuate exactly by equals amount of $(\epsilon/4) = (\phi^3/4)$. The overall (average) dimension is in this case given by the probability sums (due to the orthogonality condition among the four average dimensions) :

$$1 + \frac{\phi^3}{4}. \quad \Rightarrow 4(1+\frac{\phi^3}{4}) = 4 + \phi^3 = 4.236.... =< dim_F\ \mathcal{E}^{(\infty)} >= dim\ \mathcal{E}^{(4)} = d_c^{(4)}. \tag{25a}$$

.

After the breakdown of isotropy, one has 3 spatial dimensions, each of which could fluctuate by equal amounts, given a net spatial dimension of $3(1+(\epsilon/3)) = 3+\epsilon$. The emerging temporal dimension plus fluctuations is $1+\epsilon$. Due to the isotropy-breakdown, the temporal dimension fluctuates as much as the 3 spatial dimension combined $\epsilon = 3(\epsilon/3)$.

The long-distance coarse-grain averaging process, is tantamount of "projecting " the effective average $4+\phi^3$-dimension onto the four-dimensional smooth outer surface, which we perceive as our reality. Upon performing such projection, the intial orthogonal directions will be entangled, in the same way that a typical knot appears upon projecting it from three to two dimensions. Therefore, the spatial set and temporal set are no longer disjoint : orthogonal , as they were previously in eq-(25), they become entangled.

It is for this reason that we can experience all of the three-spatial dimensions, at any given fixed moment of clock time. One can foliate, at least locally, not globally, the four-dimensional spacetime into 3-dimensional leaves at any given fixed values of the clock time. Meaning that measurements can take place in extended space at one fixed moment of time [30]. We perceive all three spatial directions at the same moment of time. And on the contrary : It is hard for a macroscopic observer to perceive an extended interval of time, within a given region in 3-space. as Wheeler put it : " time exists to prevent everything from happening at once ". We wish to add : " space exists to prevent everything from happening at one point ".

Microscopic Particles can move forward and backwards in time because they simply have acess to more dimensions than us; i.e. particles are fully aware of the fractality of spacetime. They do not "see" the two slits in the two-slit experiment as two "separated" points : from the particle's perspective, spacetime is a discrete, fractalike Cantor set. When one constructs the Cantor set by removing the middle segment $ad\ infinitum$, one is literately " removing " the space between the fractal dust points, and hence, there is no distance between the two-slits. Because there is no distance among the fractal dust points, the Cantor Set has zero measure = zero length. Our mind is the one that fills in the void creating the illusion that there

is a distance between them. And hence the apparent " non-locality " and paradoxes of QM. Nature does not abhor the vacuum : on the contrary, it is our mind which does so by filling in the voids. Nature is quantum, discrete, it is fractal.

Therefore, the joint probability that one temporal dimension **co-exists** simultaneoulsy, with three-spatial dimensions, once fluctuations in the dimensions are properly taken into account, within the framework of Cantorian-Fractal Spacetime is given by the product :

$$(3+\epsilon)(1+\epsilon) = 4 = 3+1 \;\Rightarrow 4+\epsilon = \frac{1}{\epsilon} \;\Rightarrow \epsilon = \phi^3. \tag{25b}$$

Once again we get the Golden-Mean as an essential ingredient of reality. The true "dimensional zero-point fluctuations " about what we perceive as three spatial and one temporal dimension, at any moment of our daily experience, are in fact $\epsilon = \phi^3$. It is true than one could try, for example, writing $(5+\epsilon_1)(1+\epsilon_2) = 6$ for an infinity of values of ϵ_1, ϵ_2. However to have that $\epsilon_1 = \epsilon_2 = \phi^3$, in four, and only four dimensions, is another story....Numerical coincidence or design ?

From the lattice point of view what happens is the following : Given a quasi-ergodic self-similar fractal grid/lattice, or a fractal sphere (topologically equivalent), of dimensionality $4+\phi^3$, the coarse-grain averaging process will break down the "translational " symmetry from

$$T[4+\phi^3] \to T[4] = T[(3+\epsilon)(1+\epsilon)]. \quad \epsilon = \phi^3. \tag{25c}$$

Concluding, what we perceive, on average, as being $3+1$ dimensions is in fact the long distance coarse-grain effect or the joint " probability " overlap of the co-existence of $3+\phi^3$ (spatial) and $1+\phi^3$ (temporal) dimensions. We must recall that the set $\mathcal{E}^{(4)}$, whose dimension is $4+\phi^3$, is "packed" inside a four-dimensional sphere. Like the grains of sand on the beach : they sand "looks" two-dimensional to us, on the surface, when in fact it is 3-dimensional. Hence, as macrosocopic observers, we can only perceive the "skin " , or outer surface of the " onion " of this world. On average, on the surface, we experience a $3+1$ dimensional (where space and time co-exist) world due to a coarse-graining effect. We only perceive the **projections** onto the " four-dimensional external skin or surface " of an average $4+\phi^3$ dimension of a truly infinite-dimensional world lying underneath our " feet ", and above " our heads ". The four-dimensional "skin" , encloses the exact **average value** of $4+\phi^3$.; whose average dimension is taken over the infinite dimensions of the sets $\mathcal{E}^{(i)}$ of Cantorian-Fractal spacetime, ranging from $-\infty, +\infty$.

If we dare to " dive in " into the transfinite continuum of Cantorian-Spacetime we will be able to unravel the true fractal average dimension of spacetime, at each layer/ladder of the RG flow. There are no such things as " points " in Nature due to the Planck scale being the minimum distance in Nature. From the Wilson, Fischer, Kadanof...RG approaches to QFT, spin systems...one can infer that the Universe, upon reaching the metastable RG fixed point of $4+\phi^3$, it will signal the beginning of a dimensional phase transition, from $4+\phi^3$ to ϕ^3. One has conformal invariance at that fixed RG point $4+\phi^3$: the world , at that point, will be then self-similar at every scale (conformal invariance).

Einstein's Gravity and all the other fundamental forces in Nature are effective theories only : they all emerge from a deeper theory : the New Relativity in Cantorian-Fractal Spacetime. Vacuum fluctuations in Cantorian-Fractal Spacetime generate the long distance Einstein's Gravitational theory and all the other fundamental forces residing inside spacetime. In the conclusion we will write down the quantum master action, orginally outlined in [1,3], of a the master field in living in **C**-space, and outside of spacetime, whose vacuum fluctuations generate classical spacetime, gravty along with all the other fundamental forces. We honestly believe, that for this reason, attempts to quantize Einstein-Gravity have been futile so far. As Finkelstein has pointed out : One cannot get to Quantum Physics starting from Classical Physics. It has been tragic mistake. Cantorian-Fractal Geometry is the bridge connecting both worlds : Noncommutative Geometry in conjunction with the RG approach [11] are our mathematical/physical tools to probe into this Cantorian-Fractal world. Wavelet Analyis has been used sucessfully for Renormalization Group techniques [32]. Wavelets will also be an essential tool in the near future to unravel the multi-scaling/multi-fractal/multivector nature of the world.

The most important conclusion of this work is the following :

General Relativity required four-dimensional Riemannian Geometry for its foundations. The New Relativity requires the transfinite continuum of the Noncommutative Von-Neumann's Cantorian-Fractal Space-

time $\mathcal{E}^{(\infty)}$ [7-9]. What we perceive as a smooth four-dimensional topological space (a sphere for example) is an illusion. There are no " points " in this New Relativity, due to the fact that the Planck scale is the minimum distance in Nature: As we zoom-in with the microscope of the Renormalization Group, we uncover that each point is itself a four-sphere, and that each point within that sphere is itself four-dimensional ...*ad infinitum* : every point is truly a four-dimensional sphere.

For further connections to Donaldson exotic R^4 and the plausible geometric orgins of chiral symmetry breaking we refer to [9]. Since we have a truly Noncommutative Geometry in this Cantorian-Fractal spacetime, living underneath a smooth four-dimensional sphere, there are many different (uncountable) infinite ways to define a a Noncommutative discrete differential calculus, similar to the type formulated by Dimakis and Muller-Hoissen, Castellani and many others. Not surprisingly, one will have four-manifolds homeomorphic to R^4 ; i.e that are topologically equivalent but which are **not** diffeomorphic to R^4.

Similarly, it is possible that an internal semi-simple $SO(4) = SU(2) \times SU(2)$, symmetry could be broken by the fractal fluctuations of the dimensions down to the $SU(2) \times U(1)$. Which is in essence the philosophy behind Connes Noncommutative Geometric approach to the Standard Model of Particle Physics. There is no Higgs field introduced in by hand : it is already part of the Noncommutative geometry. Yang-Mills, in Connes view, is a " Noncommutative " fluctuation of the geometry. The New Relativity pushes this idea even further : " Classical, Spacetime, Gravity, and ALL the other fundamental forces are all fluctuations of the deeper Cantorian-Fractal Geometry ". The relevance of these ideas to Seiberg-Witten's theory of four manifolds ought to be investigated further. We are optimistic that it is the coarse-grain long-distance averaging process, of the underlying Cantorian-Fractal Geometry, that is also responsible for the chiral symmetry breakdown in Nature. The reader may ask : What happended to Supersymmetry ? The Clifford-algebraic formulation of the Noncommutative **C**-space takes care of the spin-statistics theorem.

5. The Solution to the Cosmological Constant Problem

We will present a solution to the cosmological " constant" which parallels Nottale's derivation based on Scale Relativity. Depending on the values of the "constants" a, b, or initial conditions, one will have several different cosmological scenarios. We write 'constants" because in reality these have an explicit scaling " temporal " dependence consitent with the RG approaches to the nonlinear dynamics associated to chaos, percolation, non-equilibrium critical phenomena, phase transitions self organized criticality,.....

The parameters a, b will flow in **scaling** time, t. We set the scaling initial time to be $t_o = 1$, since $ln(t_o) = 0$.

We have :

$$a(t) = (\frac{E_o}{KT})(t) = (\frac{\mathcal{E}}{KT_P})^{(t/t_o)^\alpha}. \quad \Rightarrow \frac{log\ [E_o/KT]}{log\ [\mathcal{E}/KT_P]} = (t/t_o)^\alpha.$$

$$b(t) = (\frac{R}{\Lambda})(t) = (\frac{\mathcal{R}}{\Lambda})^{(t/t_o)^\beta}. \quad \Rightarrow \frac{log\ [R/\Lambda]}{log\ [\mathcal{R}/\Lambda]} = (t/t_o)^\beta \tag{26a}$$

And for the average values of the dimensions we will have :

$$\frac{< D(t) >}{< D(t_o) >} = (\frac{t}{t_o})^{-\gamma}. \quad \Rightarrow \frac{\partial ln\ < D(t) >}{\partial ln\ t} = -\gamma. \tag{26b}$$

Equations (26b) means that the average dimension flows monotonically with scaling time. Whereas eqs-(26a) imply a power-scaling law for the ratios of the fundamental constants in Nature. Nottale's Scale Relativity assigns also a resolution-dependence to the scaling coefficients as well. To simplify matters we shall assume that the scaling coefficients are universal constants. In the section where we discussed the entropy-area relationship, we had the situation where the slope dS/dA diverges between $D = 4$ and $D = 5$. This is the signal of a dimensional phase transition : when the universe hits $< D >= 4 + \phi^3$ and begins its descent into the quasi-crystal phase of $< D >= \phi^3$. ($2 + \phi^3$, with -2 as zero reference point). To study this in full detail one requires Nottale's Scale Relativistic resolution-dependent coefficients α, β, γ , where at the critical point, one will have the desired relationship among the three coefficients. This is typical of critical phenomena, a sign of universality.

For example if the Vaccum energy was intially, $\mathcal{E}$, and of the same order of magnitude as the thermal Planck energy KT_P, at the Planck era, in general its value will flow (change) with the RG flow to a much smaller value E_o (today), compared to $\mathcal{E}$. However, it will be large in comparsison to the thermal energy observed today, $2.76 - 3$ degree Kelvin. In this New Relativity it only make sense to speak of relationships and ratios : It is a Machian Relativity : the very small is deeply interconnected with the very large. Only the relations among these values is what counts. The Universe Self-organizes in such a way that the bulk values of its (moduli) parameters are adjusted in terms of their limiting boundary values. In a sense this is once again the holographic principle acting. Information-Entropy charges flow to the boundaries, like charges in a conductor, and simply carry with them the bulk information !

The very large self-organization, from scale to scale, was a product of the continous ongoing self-recursive process that is the Universe. The Quantum Universe is a **q** process. It is the Ultimate Quantum Machian Computer. Roughly speaking, the Quantum Universe is a Self-Recursive/self-tuning/self-organizing. " organism ". For this reason the cosmological constant " problem " is never an issue in this view of the world. And in this section we will prove it.

The quantities $\mathcal{E}, \mathcal{R}$ are the vacuum energies and sizes of the original baby universes at the " initial" scaling time of $t_o = 1$, where we " begin to count ". We set our scaling " clocks " to start ticking at that moment when the radius of the bubble is $\mathcal{R}$ and the value of the vacuum energy, at that moment, is $\mathcal{E}$. Any infinitesimal perturbations $\mathcal{R} = \Lambda + \epsilon$ and why $\mathcal{E} = kT_P + \epsilon$ will continue to grow unrelented .

Physically what happened is the following. The vacuum fluctuations about the perfect balance conditions $(R/\Lambda) = (E/kT_P) = 1$ were initially infinitesimal small. However, due to the fact that one has a system with an infinite number of degrees of freedom, these fluctuations could not have been neutralized away and equilibrium restored. Because the system has infinite number of degrees of freedom, infinite dimensions, equilibrium cannot attained. Had the number of degrees of freedom be finite, then those infinitesimal perturbations would have been neutralized and equilibrium would have been restored.

The universe was driven out of balance, out of the metastable state of average dimension $4 + \phi^3$, to states of higher and higher average dimensions, until it reached a **top** average dimension , depending on the initial perturbations and the non-linear dynamics of the system. After hitting the top average dimension , the universe began its descent in average dimension towards the initial value of $D = 4 + \phi^3$.

Average dimension is a measure of the energy content of the universe. In coventional physics one assumes the the fundamental constants do not change with time. In the New Relativity they do. In particular, the speed of light c, Boltzmann and Planck constant $k, \hbar$ do change with the RG flow. They are energy dependent : $k_{eff}(E); \hbar_{eff}(E)$. With the provision that they leave the value of the Planck length invariant along the way. The Planck scale is the true " relativistic " invariant. The Planck temperature T_P is taken to be the maximum T in Nature (thermal Relativity). As the initial perturbations drove the average dimension to a higher value than $4 + \phi^3$, reaching a peak dimension, one may be inclined to think that the temperature will also rise to higher values than T_P. This is not correct due to the energy dependent contributions to the effective values of the Boltzamann constant $k_{eff}(E)$ and Planck's $\hbar_{eff}(E)$. Both values increase in such a way that the temperature does not exceed T_P. For example, a very energetic photon of frecuency ω being absorbed from the walls of the reservoir, cannot exceed the temperature bound of : $(\hbar_{eff}\omega/k_{eff}) = T_P$.

Also we have argued before, that quantum-dissipative effects during this intial chaotic phase re-heated the universe while it was expanding, and in this way the T_P was maintained during this phase. After the universe reached its peak average dimension, it began to roll down, in both dimensions and temperature : the ordered-phase began. After the universe hit its peak average dimension, it began to " roll down " The RG local scaling arrow of time began at the moment that the universe started its descent back to the metastable point, signaling the beginning of the ordered phase. Prior to that there was no arrow of time. When the universe was in perfect balance, nothing flowed, nothing changed, nothing " happened ". Spacetime as we know it did not exist. Spacetime emerged, was born afterwards. The Universe began due to a non-equilibrium process of self-organization as Prigogine has advocated in the past.

The radius of the universe is directly correlated with the RG scaling time as we indicated at the begining. One must not confuse the volume of the spheres with their radii. As we have shown, the volumes are functionally dependent on both the dimensions and the radii. Dimensions flow from larger to smaller but the radius always grow in accordance with the RG scaling time that grows monotonically. One can have an huge radius (larger than the Hubble radius) but a sphere of **zero** size in $D = -2$ and in $D = \infty$. Hence,

we must not confuse the monotonically growing radius (arrow of time) with growing volumes. They are not directly correlated.

As the Universe flows from higher average dimensions, higher vaccum energies , to lower dimensions, lower vacuum energies, information is lost. Since the **total** entropy of the universe plus reservoir cannot decrease, this means that entropy flowed out of the universe into the reservoir which is comprised by the infinite family of bubbles of all possible dimensions and radii. The quanta of spacetime create their very own background in which all the p-branes live in : the background is self-referential, it is self-supporting.

One is not violating the second law of thermodynamics. The information entropy which flows out of one universe into the self-referential thermal backgound, is itself recycled over and over again , due precisely to the bootstrap principle : all quanta/bubbles are made of each other. However this recycling process of information-energy-dimensions bootstrapping , in creating the fundamental particles in the universe , and its life forms. occurs in a hierarchical multi-fractal fashion. Self-organization occurs in discrete jumps of one scale to a larger scale to a larger scale and so forth . This is precisely the physical meaning of the transfinite continuum of the Cantorian- Fractal models of micro-spacetime Never in our wildest dreams could we possibly count the dimensions of Quantum Spacetime. Precisely because one has uncountably infinity of dimensions, one can store an uncountably infinity of information, that conscious life emerged in the Cosmos. We subscribe to Penrose's view that consciousness is a non-algorithmic process. The fundamental theory must account for consciousness in the world and this requires uncountably infinite dimensions for spacetime. $D = 11, 12, 13, 14, 26, 27$ simply will not fulfill the task. For this reason, when we speak of " dimensions " we must refer to average dimensions.

As **local** scaling time flows the values of the " constants " will change with " time " as Dirac pointed out long ago. This is consitent with the modern Variable Speed of Light Cosmologies that are becoming popular today. When one chooses the simplest case : $\alpha = \beta$ and $\mathcal{E}/KT_P = \mathcal{R}/\Lambda$. Dividing the top equation in (26a) by the bottom one yields :

$$\frac{(E_o/KT)}{(R/\Lambda)}(t) = [\frac{\mathcal{E}/KT_P}{\mathcal{R}/\Lambda}]^{(t/t_o)^\alpha} = 1. \tag{27}$$

This means then that for all values of RG scaling time, in this simple case :

$$(\frac{R}{\Lambda})(t) = \frac{E_o}{kT}(t). \tag{28}$$

For example, the exact value of the vacuum energy of the universe in this simple secenario, right now, at $R_{Hubble} = 10^{60}\Lambda$, would be :

$$E_o(today) = (kT_H)\ (\frac{R_H}{\Lambda})(today) = 10^{60}(kT_H). \tag{29}$$

This means, in this model, that the vacuum energy today would be **huge** compared to the thermal energy of T_H. This could be a very natural straighforward explanation of why the Universe today is expanding much more rapidly than people expected. Fact that baffled the whole Astrophysics community but which is a natural outcome of the New Relativity Theory and Cantorian-Fractal Spacetime. This could be a positive signal that the New Relativity/Cantorian-Fractal Spacetime view of the world may be correct or in the right course. .

The T_{COBE} value of $2.76-3$ degrees Kelvin is obtained looking into the past of the universe. The photons we detect today were emitted from the past and were redshifted due to the expansion of the Universe. Since the speed of light in the New Relativity changes with the RG flow, to calculate the exact values of the redshifts is not going to be an easy task. The correct way to do this will be to integrate backwards in scaling time along the RG trajectory, like in ordinary fluid mechanics using convective derivatives, to find out what was the true frequency these photons had upon emission.

By past meaning " the Big Bang " . There was no Big Bang in our cosmological scenario. Our model implies an aver expanding Universe since the moment the infinitesimal perturbations drove the Universe out of balance. By "expanding" we mean one whose radius flows in synchronicity with the RG flow (the true arrow of time). As said above, dimensions change in the process, and in doing so, affect the volume size even for a fixed radius. To have the volume of the spheres as indicators of temporal evolution is not a good idea. The radius is the appropriate indicator of change.

In the same fashion that the radius is an indicator of the RG flow, the COBE data's temperature T_{COBE} may be also the appropriate " thermometer " of the evolving ensemble of an infinite number of bubbles/universes. For this reason, in this model, one could set $T_H = T_{COBE}$; and conclude that the vacuum energy today, in this model, would be of $10^{60} K T_{COBE}$.

Suppose it is not enough to know what is the value of the vacuum energy at this moment , but we want to know how fast is it changing at this moment. We simply take the COBE data, calculate the " area " under its black-body curve (perform the integral) and equate the value :

$$\int L_{COBE}\ R_H^2\ d\Omega\ d\nu = Ergs/second. \tag{30}$$

and this will yield the precise value of the rate of change of the vacuum energy at this moment.

A word of caution : we must not confuse the true " local scaling " RG flow arrow of time with the ordinary reading of our clocks coordinate time. H. Kitada using Godel's incompleteness theorem has proven that a **local** time exists despite the absence of global time [33] in traditional quantum cosmology . Kitada has also arrived, using scattering theory, at the same conclusions that an infinite dimensional quantum spacetime is an essential ingredient of reality [34]. This picture is also consistent with Prigogine's ideas of persistent interactions that modify our naive views of Scattering theory.

The RG scaling time flows slowly compared with the coordinate time of our clocks. One is logarithimic the other is roughly linear. The fundamental constants of nature change with respect the RG scaling time, and for this reason they change very slowly compared to our daily experience. The speed of light when Einstein formulated his theory of Relativity is essentially the same as today. However, this was not the case when the Universe began its descend at the Planck era. The fundamental constants then changed more rapidly. We are living now in a metastable phase and for this reason life was possible in this stage of evolution. It would be hard for life forms to emerge in a period where the fundamental constants change rapidly relative to the RG scaling time flow , which is tantamount of saying that the information-entropy is no longer a linear function of the area, as we discussed previously. We live in a linear world where information-entropy transfer per unit area is constant. This is another reason why $D = 4, 5$ are optimum dimensions, consitent with $D = 4$ being an average dimension.

The COBE data would then be our " thermo-meter " that will tell us how the vacuum energy is changing. It will change very slowly from our (linear time) perspective due to logarithmic nature of the scaling time associated with the RG flow. Since the Universe is expanding much more rapidly than it was assumed, one would have to check the COBE data to see deviations from the 2.76 Kelvin, due to larger redshifts produced by a greater expansion : The Universe is **cooling** . This picture is consitent also with the RG flow towards the critical (metastable) point of (average) dimension eigenstate of the world $d_c^{(4)} = 4.236.....$

The universe can be thought as an atom which progresively cascades downwards to lower and lower energy eigenstates. Its vacuum energy progressively decreases with respect to the initial value at the Planck era. This would entail that light of greater and greater frecuency should be emitted. However, due to the continuous expansion of the universe, what should have been the emission of photons of higher frecuency , are instead redshifted due to the expansion, to lower and lower frecuencies. This implies then, that the ratio of the vacuum energy to the thermal energy, at any given moment, increases steadly. This is precisely what we find below, and we have explicit numerical calculations to substantiate our claims. The COBE data, in this model, would be the thermometer that tells us what is the temperature of the universe ensemble. Since everything flows with the scaling RG arrow of time, the COBE data must change with scaling time. Although very, very slowly from our perspective.

The New Relativity and Cantorian-Fractal Spacetime views of the world are extremely powerful tools because we can then calculate physical quantities precisely. In general we do not have $\alpha = \beta$ nor $(\mathcal{E}/KT_P) = (\mathcal{R}/\Lambda)$. To simplify mattters we shall assume that those are the conditions on the parameters.

The cosmological constant λ today , a $D = 4$ world, has units of a $(length)^{-2}$ because it must coincide with the dimensions of curvature appearing in Einstein's equations. The units of λ are precisely that of a string tension, energy per unit length. Let us now compare it with the cosmological " constant " at the moment when the click of the scaling RG time started, at the Planck era, by simply evaluating the ratios of the amounts of energy/length . It is true that in the beginning of the universe the large scale structure of spacetime (as we know it) did not even exist. Spacetime was born at that moment . Einstein's equations

do not apply then, it is a truly quantum gravity period. Nevertheless we can compute numbers .

Setting the vacuum energies to be of the same orders of magnitude as their Compton energies : $E_{today} \sim \frac{1}{R_H}$ and $\mathcal{E} \sim \frac{1}{\Lambda}$ and using eqs-(27-29) one arrives at the ratios of the cosmological " constants " :
:

$$\frac{E_{min}/\mathcal{L}}{\mathcal{E}/\Lambda} = (\frac{\Lambda}{\mathcal{L}})^2 = \frac{T_o}{T_P}. \quad (31).$$

For example, and for the sake of the argument, let us assume that one could take the upper Nottale scale to be $10^{61}\Lambda > 10^{60}\Lambda$. Later we shall see that this is not actually the case. This scenario will entail that the ratio of the cosmological " constants " will be :

$$\frac{E_{min}/\mathcal{L}}{\mathcal{E}/\Lambda} = (\frac{\Lambda}{\mathcal{L}})^2 = 10^{-122} = \frac{T_o}{T_P}. \quad (32)$$

This result implies that the cosmological " constant " , when the world was of the Planck radius (a mini-blackhole) was **122** orders of magnitude larger than the cosmlogical " constant " at the maximum radius of $\mathcal{L}$. We wish to emphasize once more, that we are referring to the radius of the universe. Since in general, dimensions change , it is not proper to compare volumes at different epochs.

Also, we would hit an extremely cold world of minimum T_o **almost zero** but **not absolute zero** . Having zero as lowest T will contradict having T_P as its **dual** maximum temperature since $1/0 = \infty$. This is the Noncommutative quasi-crystal phase of the coldest world whose $d_c^{(-2)} = \phi^3 = 0.236.....$ It may well be. This means that as we hit the RG metastable point of $D = 4.236...$ the Universe will eventually, very slowly, monotonically, exhibit another phase transition and roll slowly down into a very cold world whose lowest T_o will be **122** orders of magnitude smaller than the T_P : of the order of $10^{35}.10^{-122} = 10^{-87}$ Kelvin .... but still it will **not be** absolute zero.

To sum up : The ratio of the final minimum vacuum energy to the initial maximum vacuum energy value, consistent with the Duality principle, is :

$$\frac{E_{min}}{\mathcal{E}} = (\frac{\Lambda}{\mathcal{L}}) = 10^{-61} = (\frac{T_o}{T_P})^{1/2}. \quad (33)$$

Another way of relating the duality principle with the assumption that the vacuum energy is inversely proportional to the scale $E \sim 1/R$ is the following : Take the result from eq-(28) which says that

$$\frac{\mathcal{E}_{min}}{KT_o} = \frac{\mathcal{L}}{\Lambda}. \quad (28)$$

at close look, this seems at odds with the idea that energy is inversely related to the scales. eq-(28) is saying that energy is proportional to scales. However, this is fine because after performing the duality transformation in eq-(28) leaving the value of $\mathcal{E}_{min}$ untouched :

$$T_o \to T_P. \quad \mathcal{L} \leftrightarrow \Lambda \ \Rightarrow \frac{\mathcal{E}_{min}}{KT_P} = \frac{\Lambda}{\mathcal{L}}. \quad (34)$$

we see immediately that eq-(34) is compatible with the idea that $\mathcal{E}_{min} \sim 1/\mathcal{L}$. Furthermore, dividing eq-(28) by the eq-(34), obtained after the duality transformation, yields automatically eq-(32). So consistency is mantained all along. When we apply this same duality argument to a bubble of Planck radius, and Planck T_P, we find that the Planck scale is " self dual " in the sense that the Compton wavelength associated with the Planck's M_P, agrees precisely with the value of its Schwarzchild radius : $R \sim M_P$. This is precisely how the Planck scale is defined to begin with.

There is no such a thing as a cosmological constant " problem " in our theory because one of the consequences of the New Relativity is that the fundamental constants change with the RG scaling time : they flow with the RG flow. The world self-tunes itself. There is no need to to do any fine tuning to $60 - 61$ decimal places ! Self-Organized Critical Phenomena does it for us [16].

We have argued why the Universe will keep expanding until it reaches the upper Nottale scale, asympotically , as it reaches the final quasi-crystal phase. . Scale Relativistic corrections forbid surpassing such scale. This is the natural Infrared Regulator like the Planck scale was the natural Ultaviolet one. This is

nothing but the UV/IR entanglement mentioned earlier. For this to happen the Universe has to cool off almost to zero temperature, not the absolute zero, since this lowest T_o must be **dual** to the maximum T in Nature which is the T_P (thermal Relativity) . This entails that the Universe goes from the minimum *Planck* scale to the Nottale's upper maximum scale $\mathcal{L}$ while going from T_p to the minimum dual T_o.

The Universe can hover for enough time in the metastable state, suitable for conscious life to emerge , until the whole ensemble of infinite quanta/bubles/universes, begins its **collective** slow descent towards the lower (true vacuum) stable state of average dimension $\phi^3 = 0.236...$, or $2 + \phi^3$ relative to the $D_o = -2$ of extremely cold temperatures, a quasi-crystal , and lowest value of energy density. This quasi-crystal will grow to an enormous size, and will flow towards the regime of the minimal energy densitiy configuration. This is what eq-(32) is saying : the final value of the cosmological " constant " will be dramatically smaller than what it was at the birth of the Universe. And this will be the " end " of spacetime, matter, energy,....and life.

It is very plausible that the cold quasi-crystalized ensemble will " collide " collectively (in **C**-space, outside spacetime) with another ensemble of much higher vacuum energy, higher temperatures, a " multi-verse", and the world will begin to reheat again and climb up the dimensional ladder, to reach the metastable state of $D = 4 + \phi^3$ once again. Vacuum fluctuations will trigger another different Universe whole-life-cycle and the process could go on and on and on, for ever. It is plausible that this " Cosmic Dance " has been going on for ever and it will continue on for ever within a **cyclical** scaling time associated with the RG flow. This cyclical RG scaling time is the true universal arrow of time. The time in our clocks is just a coordinate than can be traded of for another due to the Diffemorphism invariance of General Relativity. The (cyclical) scaling RG time is the true universal time. For it to exist **negative** dimensions are crucial. A more rigourous picture of this cyclical Universes scenario requires the construction of the QFT in Noncommutative **C**-spaces [1,3.17].

What tipped the universe out of its unstable equlibrium, metastable state of average dimension $4+\phi^3 = 4.236....$, bubbles of radius equal to one Planck unit and vacumm energy equal to KT_P, was nothing but the vaccum fluctuations. These fluctuations can be arbitarily huge due to the presence of an **infinity** of quanta, bubbles of different dimensions and radii which comprised the ensemble of universes prepared by the " Universal Observer ". Quantum fluctuations in field theory can be large precisely because one is dealing with systems with an **infinite** number of degress of freedom; like our model of an infinite-dimesnional quantum spacetime. The Universe is flowing at this moment, from one regime of intial very large energy densities to another regime of lower energy densities,

This picture is not so farfectched if we remember how the Van der Walls forces, dipole-dipole intercations arise due to molecular fluctuations. The view that gravity could have risen in this way was also shared by Feynman and by Vigier and Petroni [48]. Zaharov viewed gravity as an effective theory induced by quantum matter flutuations. Our view is that classical spacetime and gravity emerged from vacuum fluctuations of the Cantorian-Fractal Geometry (dimensions, for example). Within the framework of the New Relativity and Cantorian-Fractal Spacetime, one now needs to postulate what is the " field " that fluctuates causing the dimensions to do the same. This field has been called the " Cantorion " [9]. . In the conclusion we will write down the Quantum Master Pregeometric action functional in **C**-space, outside of spacetime, that yields classical spacetime, gravity and all fundamental forces, in the long distance limit, compared to the Planck scale, from the fluctuations of the Cantorian-Fractal geometry. We get " everything " from (a Noncommutative) Geometry fulfilling Einstein's dream. Sidharth [39] has recently made some remarks about the importnace of fluctautions in Cantorian-Fractal spacetime.

Our results on why there is " no cosmological constant problem " , in the New Relativity, agree with Nottale's arguments which explained in simple terms why there is such huge discrepancy between the cosmological " constant " measured at the cosmological scales, with the cosmological " constant " measured at Planck scales. One has Scale Relativity that takes into account how measurements in Nature depend on the **scale-frame** of reference one lives in. In the New Relativity Theory , there is no such a thing as the cosmological " constant problem " if one realizes that all the contants in Nature RG flow with scaling time. And that due to a process of self-organization, self-tuning, all the constants in Nature properly adjust themselves to the RG scaling flow , or arrow of time. Such arrow of time emerged precisely because the world began out equilibrium. We agree with Prigogine.

6. Exact Evaluation of the Nottale's Upper Scale In Nature

We will evaluate shortly what is the exact value of such minimum non-zero temperature. We will see that is directly related to the maximum upper scale in Nature : the **dual** to the Planck minimum scale. Nottale gave estimates of what this scale should be. Since he was not aware of the power of the Duality principle that we learnt from string theory, he had no way of knowing how to calculate such upper scale from first principles. We do, as we will show below. This duality between large and small is the essence behind the UV/IR entanglement of QFT in Noncommutative spaces . If the upper scale were infinity, and the lowest scale were zero, there would **not** be such entanglement possible. The spacetime coordinates of a Noncommutative space do not commute because the Planck scale is not zero. ; i.e $[X^i, X^j] \sim \Lambda^2$. This property results naturally in the New Relativity : one has Clifford-algebra valued **X** multi-vectors (matrices), instead of ordinary " point " coordinates x^μ . Since **X** are Clifford-algebra valued matrices, they naturally do not commute.

The Planck scale is the ultimate UV regulator and the Upper scale is the ultimate IR regulator also. Including Nottale's Scale Relativistic corrections, meaning that if resolutions of the physical aoparatus are taken into account as extra degrees of freedom, the spheres used to prepare our ensemble will no longer be smooth and will fractalize from the very start. Their volumes will be not only radius and dimension dependent : they will be also resolution dependent as well. We will show that it is precisely when one includes Nottale's Scale Relativistic corrections (resolutions in physics) one will flow from the " metastable " quasi-ergodic state of $< D >= 4 + \phi^3$ to the truw vacuum of $< D >= \phi^3$.

Our explicit calculation have shown show that the two values of the average dimensions, using the discrete and integral in eq-(24) , was almost equal to the exact average given by the fractal dimension $d_c^{(4)} = 4 + \phi^3$, signaling the presence of a quasi-ergodic, metastable state. For this reason the world is hovering over a very long period (suitable for life) in a state whose average topological dimension is indeed very close to $D = 4$ associated with a smooth spheres (manifolds).

Finally we can compute the Scale Relativistic corrections to calculate the average dimensions. Nottale Scale Relativistic corrections essentially amount to a computation of the averages by packing the ensemble thermal reservoir " box " with fractal spheres instead of smooth ones. The average will require to integrate the values of R from R_H to $\mathcal{L}$ in the quasi-crystal phase and to integrate from Λ to R_H in the metastable phase. The former case yields the upper length in implicit form :

$$\frac{\int_0^\infty dD\ D^2\sqrt{\pi}^D[\Gamma(\frac{D+2}{2})]^{-1}\int_{(R_H/\Lambda)}^{(\mathcal{L}/\Lambda)} d(R/\Lambda)\ (R/\Lambda)^{D-1}\ [e^{\frac{E_{min}(R/\Lambda)^D}{KT_o}}-1]^{-1}\int_1^{R/\Lambda} d(\frac{\xi}{\Lambda})\ (\frac{\xi}{\Lambda})^{-D(\gamma-1)}}{\int_0^\infty dD\ D\sqrt{\pi}^D[\Gamma(\frac{D+2}{2})]^{-1}\int_{R_H/\Lambda}^{(\mathcal{L}/\Lambda)} d(R/\Lambda)\ (R/\Lambda)^{D-1}\ [e^{\frac{E_{min}(R/\Lambda)^D}{KT_o}}-1]^{-1}\int_1^{R/\Lambda} d(\frac{\xi}{\Lambda})\ (\frac{\xi}{\Lambda})^{-D(\gamma-1)}}. \quad (34)$$

The condition $E_{min}/KT_o = \mathcal{L}/\Lambda$ will give an **integral equation** that will define the upper length exactly when we set the average dimension (from the zero reference point of $D_o = -2$) to be :

$$< D' >= 2 + \phi^3 = 2.236.... = I_2[\ E_{min}/KT_o = \mathcal{L}/\Lambda\ ; \mathcal{L}/\Lambda;\ R_H/\Lambda]. \quad (35)$$

which is tantamount of choosing an average $< D >= \phi^3$. The Integral equation will **define** the exact value of $\mathcal{L}$.

In particular, Scale Relativistic corrections can shift the value $< D' >= 6.38....$ to the **precise** value of $6 + \phi^3 = 6.236$ by carefully selecting the value of the Hubble radius at the end of the metastable phase R_H, before it starts its descent into the true vacuum , the quasi-crystal phase. The temperature would be T_H (cooler than the T_{COBE}) :

$$\frac{E_{vacuum}}{KT_H} = \frac{R_H}{\Lambda}. \quad (36)$$

that must appear in the middle integrals, the Bose-Einstein integrations. Including the scale relativistic corrections, and integrating from Λ to R_H one will again have an integral equation of the form :

$$6 + \epsilon = I_1[E_{vac}/KT_H = R_H/\Lambda\ ; R_H/\Lambda] = 6 + \phi^3 = 6.236.... \quad (37)$$

that will select the exact ratios of $R_H/\Lambda = E_{vac}/KT_H$ to hit the metastable fixed RG point of $6 + \phi^3$ (relative to the zero point $D_o = -2$).

Without the use of fractal spheres and with the appropriate Nottale's Scale Relativistic Corrections one could not hit $6+\phi^3$. The latter is mesaured relative to the $D_o=-2$ which implies that $D=4+\phi^3$. Such value corresponds precisely to the fractal dimensions of a set structure living underneath a **four** dimensional smooth sphere. $d_c^{(4)}=4+\phi^3$ exactly is the Cantorain Fractal dimension of the set $\mathcal{E}^{(4)}$.

We have shown how tighly connected is the Cantorian-Fractal Spacetime model to Nottale's Scale Relativity and the use of Fractal spheres. Furthermore, using string theory Duality arguments we were able to define the upper Nottale's scale $\mathcal{L}$ exactly by means of the Integral equation that requires hitting ϕ^3 as the true dimension of the true vacuum of our theory.

It is tempting to speculate that because there are 3 different scales, $\Lambda, R_H, \mathcal{L}$ there might be a relationship among them, like a geometric mean type :

$$R_H^2=\mathcal{L}.\Lambda \;\Rightarrow (R_H/\Lambda)=(\mathcal{L}/R_H) \;\Rightarrow (\mathcal{L}/\Lambda)=(R_H/\Lambda)^2.. \tag{38}$$

Hence the two integral equations will define all the relevant scales in terms of the Golden Mean ϕ : in terms of geometry. One would have to wait to calculate the complicated integrals to establish the the values of the $R_H, \mathcal{L}$ exactly and to see whether or not they obey a geometric mean relationship (38).

7. Conclusions

The New Relativity in conjunction with Cantorian-Fractal Spacetime has allowed us to arrive at the following results :

1. We have derived in two-lines the area-entropy relationship in any dimension, and shown why the Bekenstein-Hawking linear relationship is **not valid** for all dimensions. It is only valid in a limited range.

2 . The New Relativity, in conjunction with the (Noncommutative) Cantorian-Fractal Spacetime, provides with a truly background independent formulation of quantum gravity :Spacetime does not exist *ab initio* : it is a process in the making [14] . The true quanta of spacetime are the p-loop histories where $p=0,1,2,3.....$ And also why Fermat's last theorem requires a multi-dimensional world. Godel's incompleteness theorem was essential in invoking the notion of Self-Referential background.

3 . we have proven **why** (on the average) we live in $3+1$ dimensions based on the Cantorian-Fractal Spacetime .

4. why negative topological dimensions are very important. The Cantorian-Fractal Spacetime model provides with a natural definition of negative information-entropy, or anti-entropy as argued by Conrad [38]. We take our true vacuum as one having $D=-2$ topological dimensions meaning that all the other dimensions below this one are filled up, like a Dirac sea of negative energies.

5 . we have argued how fluctuations in the Cantorian-Geometry may induce chiral-symmetry breaking in Nature.

6 . we have shown why there is no " cosmological constant problem " within the framework of the New Relativity Theory The RG flow of the fundamental constants in Nature , with respect to the local universal scaling time, (Self-tuning) automatically takes care of this. Our results agree with Nottale's elegant resolution of the cosmological constant problem.

7 . we have an alternative cosmological scenario to the Big Bang Model, the Inflationary models and the modern Variable Speed of Light Cosmologies. The Universe originated from a process of Non-equilibrium Self-Organized Critical phenomena about the initial balanced conditions of :

$$\frac{\mathcal{R}}{\Lambda}=\frac{\mathcal{E}_{vac}}{kT_P}=1. \tag{39}$$

In order for the Universe to have originated from a non-eqilibrium process of self-organization, it is essential to have an infinite-dimensional quantum spacetime and such spacetime is precisely provided by the Cantorian-Fractal Spacetime . The world is a continuous, on-going, iterated, self-recursive, non-linear, complex dynamical process which organizes itself in a multi-scaling/multi-fractal/multi-vector (Clifford algebras) fashion : in discrete jumps, from scale to scale. We agree with Prigogine's original views of how the Universe could have started as a phase transition : order out of chaos. For a book on Quantum Chaos see [37]; for the role that fractals have in QM, Information, Diffusion... , [38, 40]

8 . Today we are slowly approaching the metastable RG fixed point of $< D >= 4 + \phi^3$. It would be worthwhile to reformulate our results within the language of the Wilson, Fischer, Kadanof.....RG approach. The radius will continue to grow and the Universe will move away very slowly from the metastable state we live in, $< D >= 4 + \phi^3$, to the true vacuum of the world. It will eventually be reaching the Nottale upper impassible length when it hits the RG fixed point of $d = \phi^3 = 0.236.....$ This is the limiting true final average dimension of the world , associated with the (Noncommutative) Cantorian-Fractal quasi-crystal phase of the Quantum Spacetime. This will be the "end ", the final temperature will be almost zero, but not exactly zero.

9 . We have a very large number of possible cosmological scenarios, that depend on five parameters (four independent) : $a_o, b_o, \alpha, \beta, \gamma$. It is desirable to find their correct values that fit the astrophysical data. Among such scenarios, one can have a self-similar branched-polymer phase of the universe where an infinity of baby-universes branch-off. This is the case when $a_o > 1$ and $b_o = \mathcal{R}/\Lambda = 1$, for example.

The COBE data may turn out to be of crucial importance to check the validity of one particular model. We presented a simple model how one could tell exactly what is the (changing) value of the vacuum energy, at any moment of RG scaling time, and how fast is changing using the COBE data as our natural " thermo-meter ".

10 . For finite values of D, The Quantum Master action functional for the field $\Psi[X(\Sigma)]$ where $X(\Sigma)$ is a **C**-line in **C**-spaces, **outside spacetime** , was proposed in [1,3,17] and is of the following form :

$$S[\Psi(X(\Sigma)] = \int [DX(\Sigma)] \ \frac{1}{2} \ [\frac{\delta\Psi}{\delta X} * \frac{\delta\Psi}{\delta X}] \ + \frac{1}{2}\mathcal{E}^2 \ \Psi * \Psi \ + \frac{g_3}{3!} \ \Psi * \Psi * \Psi \ + \frac{g_4}{4!} \ \Psi * \Psi * \Psi * \Psi. \tag{40}$$

Such QFT of interacting line functionals in **C**-space is currently under investigation [17]. In a few lines :

The Quantum Group symmetry of such Noncommutative QFT is given by a Braided-Hopf-Quantum-Clifford Algebra, which are becoming very popular today. The three point vertex is given by the product and coproduct of the quantum algebra : annihilation of two **C**-lines into a third one or the creation of two **C**-lines from one **C**-line.

The four point vertex corresponds to the **braiding** of the quantum algebra : scattering of two lines into two final ones. The two-point vertex corresponds to the **pairing** of the quantum algebra (the tension/mass squared term). The analogs of "photons" in **C**-space are tensionless p-branes. The kinetic tem is the generalization of the Witten-Zwiebach open/closed string field kinetic terms, based on the Operadic Gerstenhaber Batalin-Vilkowisky Quantum Master action : an extension of the BRST quantization of ordinary gauge theories.

The Cantorion field $\Psi[X(\Sigma)]$ is a hyper-complex number or Clifford-algebra valued object. In particular, it could be quaternionic or octonionic valued. The **C**-lines in **C**-space are nothing but the Clifford-algebraic extension of Penrose's twistors based on complex numbers. The action is unique in the sense that the Braided-Hopf-Quantum-Clifford algebra fixes what type of terms are allowed in the action. The power behind such action is that it is the unique action that governs " everything " : it governs the creation of space-time itself, the origins of gravity and all the fundamental interactions living in spacetime. The $\Psi[X(\Sigma)]$ Cantorion field of Cantorian-Fractal Spacetime was responsible, due to its vacuum fluctuations, for the creation of the Quantum Universe we live in.

11 . In the $D = \infty$ limit, one may construct a Unique Topological action for the world. The large $D = N = \infty$ limit was discussed in [47] briefly, concerning the relationship among Conformally Invariant σ-models on Anti de Sitter Spaces , AdS_{2N} , and Zaikov's Chern-Simons **p'-brane** Field theories living on the projective boundaries of the (Euclideanized) Anti de Sitter spaces , S^{2N-1} spheres. When $2N+1 = D$ and $2N = p+1 = p'+2$ all go to ∞, there is **no** distinction between p and $p' = p-1$, hence Zaikov's Chern-Simons p'-brane Topological QFT (global degrees of freedom) acquires true **local** dynamics. Zaikov's Chern-Simons p'-brane QFT is the natural Topological Field Theory for the world. In this case, $2N + 1 = D = \infty$, the Chern-Simons p'-brane becomes the (infinite dimensional) spacetime filling p-brane [1].

For example, a Chern-Simons **10-brane** lives on the exotic sphere S^{11}, that is the projective boundary of the Euclideanized AdS_{12} space, which , in turn, can be embedded in a $D = 13$-dim flat spacetime. One can see the clear interplay among $10, 11, 12, 13$ dimensions. We found [47] that the instanton solutions, associated with the Conformally invariant $O(12, 1)$ σ model, correspond geometrically, to the " stereographic

maps " from the " souh pole " of the (Euclideanized) AdS_{12} space , onto the " equator " R^{12}. One has two hyperbolic branches, the projection from the ("south pole") top of the southern branch, onto the " equator " R^{12}, will intersect the northern branch at a point P'. By equator we mean the R^{12} that divides R^{13} into a north/south region.

12. Zaikov's Chern-Simons classical p-brane field theory admit W_∞ algebras as the algebra of constraints. The connection to Vasiliev's Higher Spin Conformal Field theories [44], **W** geometry and W_∞ algebras, based on Moyal-Fedosov deformation quantization, was given in [43]. When the $D \to \infty$ limit is taken, one makes contact with the transfinite continuum of Cantorian-Fractal Spacetime. Infinite-dimensional Loop Spaces and their associated infinite-dimensional Loop algebras need to be investigated. A very hard topic. The " infinite colour " limit of W_∞ algebras : W_∞^∞ is also worth pursuing. Another difficult enterprise. Afterwards, one may study the Quantum Braided Hopf algebraic extensions of those W_∞ algebras. This is going to be a very difficult task to achieve. For this reason it is easier to focus solely on the Quantum Master action (40) for the time being; i.e. breaking the infinite-dimensional Topological symmetry by truncating the infinite number of dimensions to a finite value D. One could start for example with $D = 26$. For recent work related to Holography, Moyal Quantization and W algebras see [45,46].

Concluding :

It is our belief that the evolutionary process of the Universe is intricately connected with the dynamics of self organized criticality, complex systems, self referential noise, or Liebnitz **monads**, as a fundamental aspect of reality, quantum dissipative processes. The Universe, as it evolves, it simultaneously self tunes itself. Roughly speaking, it is related to what Finkelstein has called a **variable quantum law**. Clearly the naive Lagrangian formalism in ordinary spaces will not work. The master action (40) lives outside spacetime. One needs a more general formalism. This is why we recur to the RG approach.

The RG approach is an essential ingredient of the propagation of strings in curved spacetime backgrounds. A vanishing β function for the world sheet couplings encodes the motion dynamics of the spacetime propagation of strings. i.e. The strings propagate in those curved backgrounds that are solutions to the coupled Eintein-Yang-Mills equations, for example, and the latter, can be re-expressed as the vanishing of the beta functions associated with the world sheet couplings. The *scaling* dynamics encodes the *motion* dynamics of the strings. Hence, the renormalization group process "chooses" the very own backgrounds in which the strings can propagate. The fact that there is **duality** in M theory which relates all known superstring theories in different corners of its moduli space, suggests that the Lagrangian formalism is perhaps **redundant**. Critical phenomena are related to **universality**.

To sum up , we believe that the Universe as it evolves it constructs along with it its very own Hilbert space. Hence, **nonlinear** complex dynamical systems has to be an essential part of reality. String Theory, Noncommutative Geometry, Quantum Groups, Hopf Algebras, and the New Scale Relativity, for example, already have shown that at the fundamental level, coordinates do not commute and the Heisenberg Uncertainty Relations are modified to account, precisely, for the Planck scale to be the minimal length in Nature. As we try to compress the strings, membranes, p-branes... to smaller scales than the Planck one, the strings,....begin to grow in size signaling that there is a nontrivial Ultra Violet/Infra Red entanglement. The Planck scale must **not** be confused with the string scale l_s. D-branes can probe distances smaller than l_s.

These arguments, in turn, forces upon us to abandon the Archimedean geometry and the use of real numbers for a non-Archimedean geometry and p-adic numbers [42]. The Quantum Universe is not a **thing** . It is a **process**. As a typical life organism, the Universe , the ultimate Quantum Machian Computer, self tunes itself to account for the value of its fundamental constants and for the existence of life. Evenfurther , the classical *spacetime* that we perceive with our senses is an **emerging** entity as well. It is something that does **not** exist *ab initio*, it is a *process* in the making [14]. Therefore, a more fundamental aspect of reality must start by abandoning the very notion of spacetime itself. And by abandoning it, we must also abandon with it, the Lagrangian formalism as well. It is meaningless to say that we have a field at point **x**. To physically define the mathematical entity **x** one needs to specify an infinite number of digits associated with real numbers. This is one of the main motivations to use p-adic numbers from the start in Physics [42].
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It is **essential** to emphasize that one does not have "a given " particular universe inflate to the size observed today. In a true **q process** what one has is a hierarchical family of universes, like a family of

Matryoshka dolls, and each representative member of that hierarchical family has on **average** dimension , as seen **today** of approximately $4 + \phi^3 = 4.236067977.....$ We must see our world only as a representative of an infinite **ensemble** of universes instead of " a given " and fixed universe inflating to the sizes of our universe today. This is the essence of using the ensemble density to calculate the average dimension. We are really " performing " a Feynman path integral through the infinite possible scenarios/histories of the world .

Morereover, there is a perpetual Heraclitean process goin on all the time. The world tomorrow is **not** the same world today. We are truly observers and participants, as Wheeler said, in this averaging process we perceive as reality : the Everret-De Witt-Wheeler many worlds interpreation of QM. We perform billions of measurements every day , at every second, with one of the most sophisticated measuring devises there are around : our brains [41]. Every time an observation is made, a branching of the possible secenarios will occur. It is for these reasons why we believe that the Physics of the Human Brain ought to capture a leading role in the Physics of the next century [30,41], as Penrose anticipated.

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References

1- C. Castro : " Hints of a New Relativity Principle from p-Brane Quantum Mechanics " to appear in the J. Chaos, Solitons and Fractals.

C. Castro : " The Search for the Origins of M Theory : Loop Quantum Mechanics and Bulk/Boundary Duality " hep-th/9809102

S. Ansoldi, C. Castro, E. Spallucci : " String Representation of Quantum Loops " Class. Quant. Gravity **16** (1999) 1833.

hep-th/9809182

2- C.Castro : " Is Quantum Spacetime Infinite Dimensional ? " to appear in J. Chaos, Solitons and Fractals.

3- C. Castro : " The String Uncertainty Relations follow from the New Relativity Theory " with Foundations of Physics.

4- C. Castro, Alex Granik : " On M Theory, Quantum Paradoxes and the New Relativity " physics/ 0002019

to appear in the J. Chaos, Solitons and Fractals.

5- C. Castro : " An Elementary Derivation of the Black-Hole Area-Entropy Relation in Any Dimension " hep-th/0004018

submitted to the Int. Jour. of Theor. Physics .

6- L. Nottale : " Fractal Spacetime and Microphysics : Towards a Theory of Scale Relativity " World Scientific, 1993.

" La Relativite dans tous ses Etats " Hachette Literature, Paris, 1998.

7- M.S. El Naschie : " von Neumann Geometry and $\mathcal{E}^{(\infty)}$ Quantum Spacetime " .

J. Chaos, Solitons and Fractals. **9** (1998) 2023.

8- M.S. El Naschie : Int. Jour, Theor. Physics **12** (1998) 2935.

9- M.S. El Naschie : " Nuclear Spacetime Theories, Superstrings, Monster Group and Applications " .

J. Chaos, Solitons and Fractals. **10**(2-3) (1999) 567-.

M. S. El Naschie : " Towards the Geometric Unification of all Fundamental Forces based on Cantorian-Fractal Spacetime " to appear in the J. Chaos, Solitons and Fractals .

M. S. El Naschie : " Fractal Gravity and Symmetry Breaking in a Hierarchical Cantorian-Spacetime "

J. Chaos, Solitons and Fractals. **8** (11) (1997) 1865

10- G. Ord : " Gravity and the Spiral Model " J. Chaos, Solitons and Fractals. **10**(2-3) (1999) 499.

11- D. Kreimer : " Shuffling Quantum Filed Theory " hep-th/9912290.

12- T. Gannon : " Conformal Field Theories and the Monstrous Moonshine : hep-th/

13- R. Cahill, C. Klinger : " Self Referential Noise as a Fundamental Aspect of Reality " gr-qc/9905082.

14- A. Aurilia, S. Ansoldi , E. Spallucci : J. Chaos, Solitons and Fractals. **10**(2-3) (1999) 197 .

15- T. Nowotny, M. Requardt : J. Chaos, Solitons and Fractals. **10**(2-3) (1999) 469.

16- L. Smolin, S. Kaufmann : " Combinatorial Dynamics in Quantum Grvaity " hep-th/9809161.

17- A. Aurilia, S. Ansoldi , C. Castro and E. Spallucci : In preparation

18- C. Castro : " Branes from Moyal Deformation Quantization of Generalized Yang Mills Theories " hep-th/9908115.

19- M. Agop, P. Ioannu, P. Coman, B. Ciobanu and P. Nica : " Cantorian-Fractal Spacetime and Generalized Superconductivity "

20- I. Aref'eva, P. Medvedev, O. Rytchkov and I. Volovich : J. Chaos, Solitons and Fractals. **10**(2-3) (1999) 213. .

21- R. Cahill, C.Klinger : " Self-Referentai Noise and the Synthesis of $3D$ Space " gr-qc/9812083.

22- L. Nottale : Private Communication.

23- C. Castro : Int. J. of Modern Physics **A 13** (8) (1998) 1263. .

24-J. Chaline, L. Nottale, P. Grou : " Is the Evolutionary Tree of Life a Fractal Structure " C.R. Acad. Science, Paris.

25- W. Pezzaglia : " Polydimensional Supersymmetric Physics " gr-qc/9909071; to appear in Int. J. Theor. Physics .

26-. H. Perez Rojas, R. Torres : Phys. Letts **A 137** (1989) 13.

27- G. Chapline, K. Yagamishi : " Induced Self Dual Gravity : a W_∞ algebraic approach ". Class. Quantum. Gravity **8** (1991) 427.

28- Javier Rodilla Luque : Private Communications with C. Castro at the University of Puerto Rico, Rio Piedras, July, 1990.

29- J. Ellis, N. Mavromatos, D. Nanopolous : J. Chaos, Solitons and Fractals. **10**(2-3) (1999) 345.

30- D. Chakalov : Private Communication.

31- T. Thiemann : " Quantum Gravity as the Natural Regulator of Matter QFT " gr-qc/9705019.

32-. G. Battle " Wavelets and the Renormalization Group " World Scientific

33- H. Kitada : " A Possible Solution to the nonexistence of Time " gr-qc/9910081.

34- S. Paul King : Private Communication

35- D. Finkelstein : " Spacetime Code " Phys. Review **D 5** (1972) 320. Phys. Review **D 5** (1972) 2922. Phys. Review **D 9** (1974) 2219

36- L. Kobolev : " Are the Laws of Thermodynamics Consequences of a Fractal Property of the Universe " phys/0003036.

37- K. Nakamura : " Quantum Chaos " . Cambridge University Press 1995.

38- M.S. El Naschie, O. Rossler , G. Ord : " Introduction to Chaos, Information and Diffusion in Quantum Mechanics " J. Chaos, Solitons and Fractals. **7** (5) (1996) May issue.

39- B. Sidharth : " A Brief Note on Fluctuations and Interactions " to appear in the J. Chaos, Solitons and Fractals.

40- H. Kroger : " Fractal Geometry in Quantum Mechanics, Field Theory and Spin Systems " Phys. Reports **323** (2) (2000).

41-D. Koruga : " Information Physics : In Search of the Scientific Basis of Consciousness " Edited by S. Hameroof et al .M.I.T press 1995.

42- M. Pitkannen : " p-Adic TGD : Mathematical Ideas " hep-th/9506097

A. Khrennikov : " p-adic numbers in Classical, QM...." quant-ph/0003016.

L. Brekke and P. Freund : " p-adic Numbers in Physics " Phys. Reports **231** (1993) 1-66

V. Valdimorov, I. Volovich and E. Zelenov : " p-adic Numbers in Mathematical Physics . World Scientific , Singapore 1994.

M. Altaisky, B. Sidharth : " p-adic Phyiscs below and above Planck Scales " J. Chaos, Solitons and Fractals. **10** (2-3) (1999) 167

D. Ghoshal, A. Sen : " Tachyon Condensation and Brane Descent Relations in p-adic String Theory " hep-th/0003278

43-C. Castro : Jour. of Geometry and Physics **33** (2000) 173.

44- M. Vasiliev : " Higher Spin Gauge Theories in four, three and two dimensions " hep-th/9611024.

45- T. Deroli, A. Verciu : J. Math. Phyiscs **38** (11) (1997) 5515.

46- A. Granik, G. Chapline : " Moyal Quantization , Holography and the Quantum Geometry of Surfaces " J. Chaos, Solitons and Fractals. **10** (2-3) (1999) 319.

47- C. Castro : " Conformally Invariant σ models on Anti de Sitter Spaces, Chern-Simons p-branes and W Geometry " hep-th/9906176. This paper was orginally accepted by the two referees of Nuclear Physics B. Almost 3 months later, we received a very highly suspicious declination letter due the shameful interference of an Editor, who had shown " great interest " in this work. In view of the suspicious circumstances we are inclined to believe that the Editor had a great interest in preventing this work from being published. We resist the temptation to broadcast the name of this UC.Berkeley professor of Physics. Nuclear Physics B never denied that we knew who he was.

48- J. Vigier, C. Petroni : " Markov Processes at the velocity of Light, The Klein-Gordon statistics "
Int. Jour. Theor. Physics **18** (11) (1979) 807.

49- G. Dito, M. Flato, D.Sternheimer and L. Takhajan : : Deformation quantization and Nambu Mechanics " hep-th/9602016.

50- M. Kontsevich : " Deformation Quantization of Poisson Manifolds " q-alg/9709040

51- E. Guendelmann , E.Nissimov, S. Pacheva : " Composite Vector and Antisymmetric Tensor Field Theories
of Volume Preserving Diffeomorphisms " . hep-th/9903245 .

52- Nuclear Physics B : Private Communication with a member of the Editorial Staff.

53- G. Bekkum : Private Communication

54- L. Smolin : Private Communication with Carlos Castro.